DESIGN OF CONTINUOUS LAG COMPENSATORS

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Abstract

In this paper design of continuous lag compensators with specified structure for linear system is addressed, proposed and tested. The design of lead compensators is proposed by using Bode diagram. The paper deals with theoretical and practical methodology, and its successful application for controlled systems. Advantages and disadvantages of phase lag compensators are summarized.

1 Introduction

Automation algorithms are installed in various areas of industrial production equipment, which allow self-control of processes without the involvement of humans. One of such devices is the correcting element with specified structure [1]. The phase lag compensator with specified structure is a dynamic system, which adjusts static and dynamic characteristics of controlled system. In this paper serial connection of above mentioned controllers is discussed, see Fig. 1.

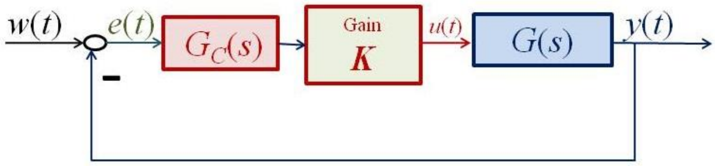

Figure 1: Control structure with compensator $G_C(s)$ – $w(t)$ is reference input signal, $e(t)$ is error signal, u(t) is input signal to the plant model with transfer function $G(s)$, $y(t)$ is plant output

The simplest structure of the phase lag compensator is the first order system with transfer function

$$G_C(s) = \frac{aTs+1}{Ts+1} \tag{1}$$

where K, T, a computed parameters, a<1.

2 Design of Phase Lag Compensator

The steady-state error can be reduced (not eliminated) by using phase lag compensator. The phase $\varphi(\omega)$ is negative and is from interval $\varphi(\omega) \in \langle -\pi/2, 0 \rangle$. Transfer function of compensator is given by (1) and Bode diagram is shown in Fig. 2.

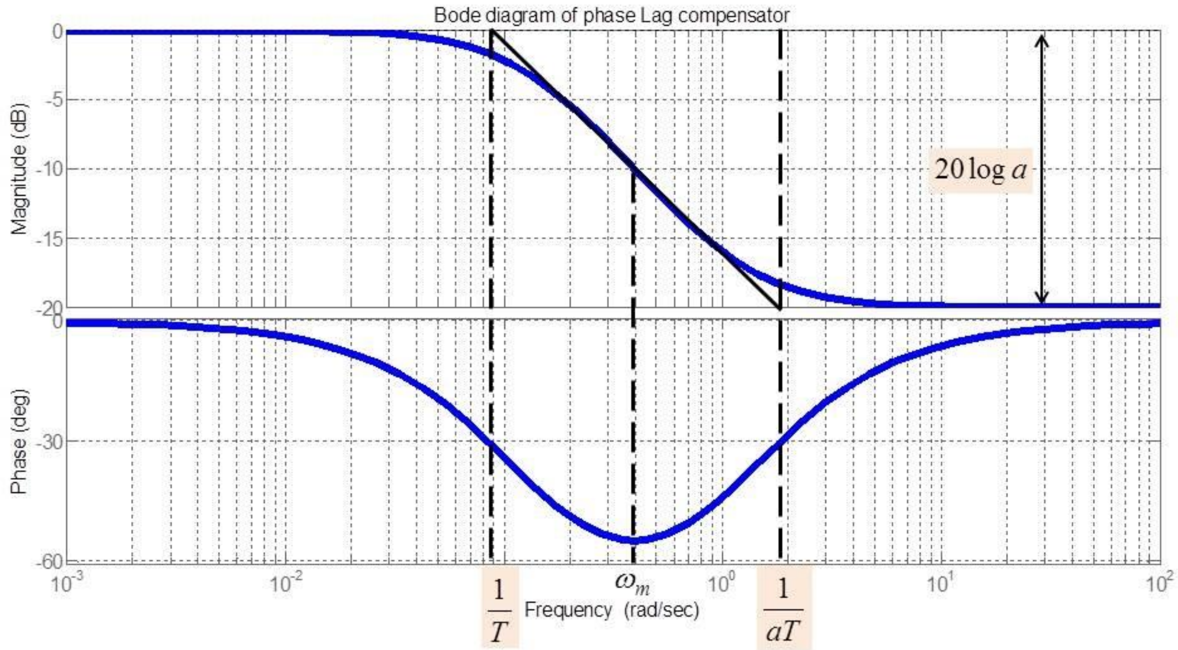

Figure 2: Bode diagram of lag compensator

Algorithm of design consists of the following steps [2]:

1. Determining of the value of gain K for open control loop. It is chosen to satisfy steady-state performance requirement.
2. For expressed value of the gain is plotted the Bode diagram of uncompensated open control loop $KG(s)$.
3. For required value of phase margin $\Delta\varphi_{0Z}$ is deduced value of the future amplitude intersect ω_a from the Bode diagram. The phase curve is changed around ω_a. So ω_a is compensated (approximately about 3-10 rad/sec) to the new gain crossover frequency $\omega_a^{'}$, where the compensated magnitude curve passes through the 0 dB axis.
4. Angular frequency $\omega_a^{'}$ will be new magnitude crossover of magnitude curve of open control loop. Is assumed that magnitude Bode characteristic of compensator in point $\omega_a^{'}$ will have amount -20log a dB

$$A = 20\log|KG(j\omega_a^{'})| = -20\log a \tag{2}$$

from where

$$a = 10^{-\frac{A}{20}} \qquad a < 1 \tag{3}$$

5. The validity of relationship (2) will be ensured by moving of magnitude curve to the left. Upper point $1/aT$ will be choosen one decade below as $\omega_a^{'}$ [Harsányi *et al.*, 1998].

$$\frac{1}{aT} = \frac{\omega_a^{'}}{10} \tag{4}$$

from where

$$T = \frac{10}{a\omega_a^{'}} \tag{5}$$

6. Constant T is now known. Design of phase lead compensator is completed. The design is checked by simulation of the step response. Plot also the Bode diagram of compensated open control loop. If performance requirements are met, stop. Otherwise go back to step 1.

3 Advantages and disadvantages of phase lag compensator

- The gain is moved crossover to a lower frequency while the phase curve is unchanged [3].

- The rise time and the settling time of the system are usually slower.
- The bandwidth of the closed-loop system is decreased.

4 Case Study and Simulation Results

Consider the following system

$$G(s) = \frac{1}{s(T_1 s+1)} = \frac{1}{s(0.0726s+1)} \tag{6}$$

The task is to design a continuous compensator that satisfies the following requirements:

1. Steady-state error $e(\infty) < \varepsilon$ due to a unit ramp function input $w(t)=t$, where $\varepsilon=0.01$.
2. Maximum percent overshoot $\eta_{\max}<30\%$ or phase margin $\Delta\varphi_{OZ}>45°$.

Solution of the example:

The gain K=100 is expressed from the steady-state error for $w(t)=t$ as follows

$$e(\infty) = \lim_{s\to 0} sE(s) = \lim_{s\to 0} s\frac{1}{1+G_0(s)}W(s)$$

$$e(\infty) = \lim_{s\to 0} s\frac{W(s)}{1+KG(s)} = \lim_{s\to 0} s\frac{\frac{1}{s^2}}{1+K\frac{1}{s(0.0726s+1)}} = \frac{1}{1+K} \le 0.01 \Rightarrow \quad K \ge 99 \tag{7}$$

The controller is now a P (proportional) controller (K=100). The transfer function of open-loop system will be as follows

$$G_0(s) = KG(s) = \frac{100}{s(0.0726s+1)} \tag{8}$$

Bode diagram for the uncompensated system $G_0(s)$ is shown in Fig. 3.

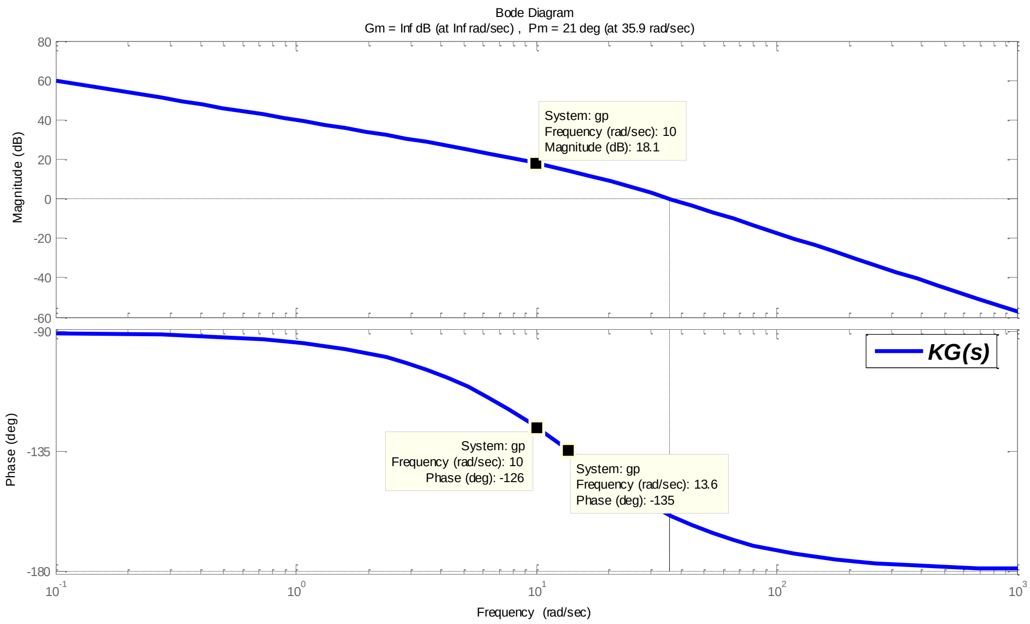

Figure 3: Bode diagram of (8)

The uncompensated gain crossover frequency (with K) is 35.9 rad/sec and the phase margin is $\Delta\varphi_{OZ}$ =21°.

From the Bode diagram in Fig. 3 is deduced the frequency of the future amplitude intersect ω_a for the desired value of phase margin $\Delta\varphi_{0Z}$=45°. For this system is ω_a=13.6 rad/sec. The phase curve is changed around ω_a. Therefore, ω_a is compensated (approximately about 3-10 rad/sec) to the new frequency $\omega_a' = 13.6 - 3.6 = 10$ rad/sec. This value will be the new crossover of magnitude curve of compensated open-loop system. From the Bode diagram is deduced the value of magnitude A=18.1 dB for frequency ω_a'. The corresponding value of a is calculated by using (3), and the result is a=0.1135. The parameter T is 8.81 according equation (5).

The transfer function of phase lag compensator from (1) is

$$G_C(s) = \frac{s+1}{8.81s+1} \tag{9}$$

and Bode diagram is shown in Fig. 4.

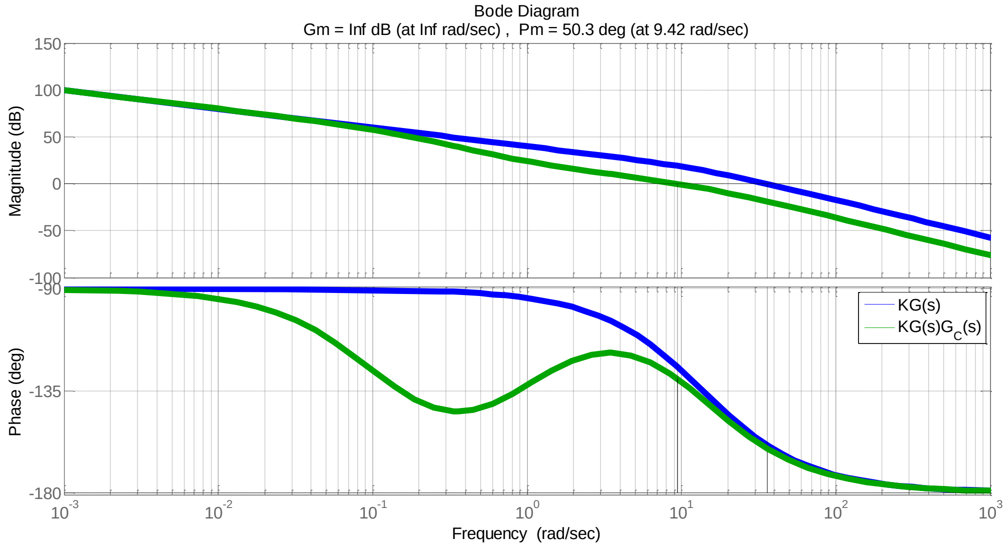

Figure 4: Bode diagram for compensated (green) and uncompensated system (blue)

In Fig. 4 the blue curves are for uncompensated system and green for system with the phase lead compensator. The compensated gain crossover frequency of 9.42 rad/sec and the phase margin Pm=50.3°. Time response of the output variable with lead compensator is shown in Fig. 5.

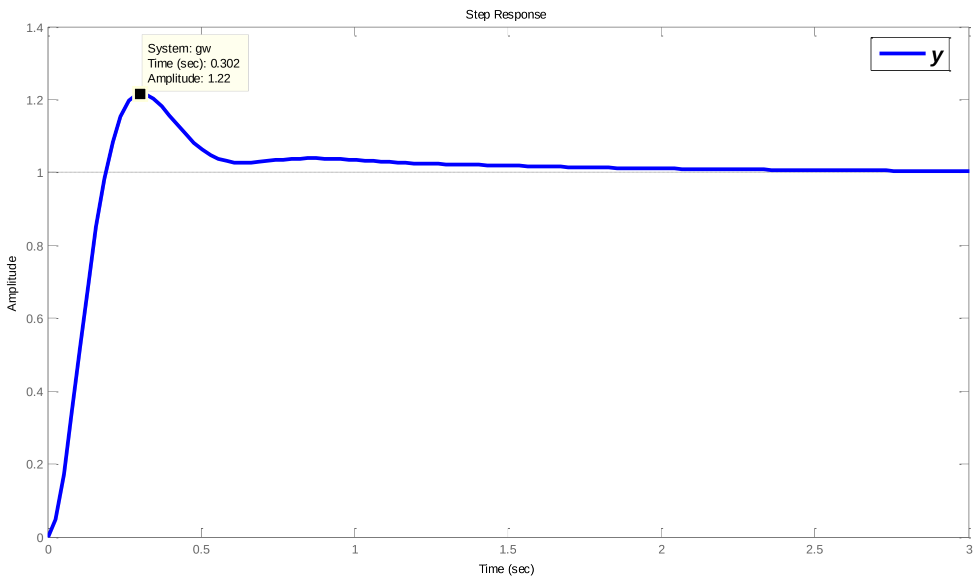

Figure 5: Time responses of the output variable with lead compensator

Maximum percent overshoot η_{max}=22%.

5 Conclusions

The specifications of the system (6) were satisfied for compensator the phase lag (9). The phase margin, steady-state error and maximal percent overshoot specifications were satisfied. The phase lag compensator adds gain at low frequencies. This compensator increases damping of system, rise time and settling time.

ACKNOWLEDGEMENT

This paper has been supported by the Slovak Scientific Grant Agency, Grant No. 1/2256/12.

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