COMPARISON OF DIFFUSION MODELS IN ASTRONOMICAL OBJECT LOCALIZATION

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Abstract

This paper is devoted to methods of objects localization based on the comparison of acquired image data and dark image of used imaging system and also focuses on modeling of detected objects. There are processed astronomical data acquired during night with long exposure times. Object localization is based on the presumption that the data acquired by CCD imaging systems are Poisson distributed. For objects localization in astronomical science are commonly used statistical models based on the different density or mass functions of probability distributions. There are usually applied Gaussian and Moffat functions, which are compared together. For purpose of models parameters optimization was used Matlab `fmincon` function.

1 Introduction

Objects detection [1] and their exact localization is one of the most fundamental topics in astronomical images processing [2, 3]. Analyzed data are usually acquired during the night when light conditions are poor. Thus it is necessary to use long exposure times.

For data acquisition, an astronomical CCD camera [4] is used. CCD sensor [4] is a source of several noises [5]. Suppose that it works as an photon counter, then it is logical that the images are contaminated by photon counting noise [2]. This is result of the fact, that the light is used as an information carrier and thus we must consider its behavior as a stream of photons.

Objects localization, as described further in the article, is based on estimation of used systems response to the the impulse, i.e., estimation of Point Spread Function (PSF). These are modeled by different diffusion models. Methods applied in this article are based on optimization of the objective function. This is derived by Maximum Likelihood Estimate [6, 7] (MLE) method and it describes relation between analyzed images and used object model.

2 Objects detection and PSF modeling

2.1 Noise Model

Astronomical images can be expressed in mathematical way as follows

$$\mathrm{x}(k,l) = \mathrm{f}(k,l) + \mathrm{n}(k,l) \tag{1}$$

where $\mathrm{f}(k,l)$ are the data and $\mathrm{n}(k,l)$ represents noise called the dark current. This type of noise is caused by thermally generated charge, due to the long exposure times. Dark current should be simply removed by a dark frame, which maps mentioned thermally generated charge in CCD sensor. It can be considered that this type of noise is Poisson distributed [1] in the following way

$$\mathrm{n}(k,l) \sim \mathrm{Poisson}(\lambda(k,l)) \tag{2}$$

where λ(k,l) is expected number of occurences in the CCD pixel cell (k, l) and $\lambda \in \mathbb{R}_0^+$. This claim can be verified on a sample of the dark images by a statistical test for the Poisson probability distribution, which can be found in [1, 8].

In the following text we will consider an average dark frame

$$\mathrm{d}(k,l) = \frac{1}{m} \sum_{i=1}^{m} \mathrm{n}_i(k,l) \tag{3}$$

where m is the number of dark images, $\mathrm{n}_i(k,l)$ is a noise in i-th frame and futher we can assume that $\mathrm{d}(k,l) = \hat{\lambda}(k,l)$.

2.2 PSF modeling

As mentioned in introduction, modeling of astronomical objects is based on estimation of used system PSF. PSF presents response of applied imaging system on the Dirac delta function, fo discrete systems defined as unit impulse

$$\delta(x) = \begin{cases} 1 & \text{if } x = 0 \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

One of the most fundamental diffusion model is a two-dimensional Gaussian function. Model image with astronomical objects described by the diffusion model can be derived from Eq. (1) by replacing the expression $\mathrm{f}(k,l)$ in the following way

$$\mathrm{x}(k,l) = \mathrm{f}(k,l,\boldsymbol{p}) + \mathrm{n}(k,l) \tag{5}$$

where $(k,l) \in \mathbb{D}^{M \times N}$, M and N are dimensions of the rectangle region of interest $\mathbb{D}$ and $\mathrm{f}(k,l,\boldsymbol{p})$ is a Gaussian diffusion model of astronomical object. The model $\mathrm{f}(k,l,\boldsymbol{p})$ is possible to express by the following relation

$$\mathrm{f}(k,l,\boldsymbol{p}) = p_1 \exp\left(-\frac{(k-p_2)^2 + (l-p_3)^2}{2p_4^2}\right). \tag{6}$$

where its parameters respectively are amplitude (p_1), coordinates in $x-y$ plane $(p_2,\ p_3)$ and standard deviation (p_4). The second commonly used PSF model was described by Moffat [9], which a special case of Cauchy distribution

$$\mathrm{f}(k,l,\boldsymbol{p}) = \frac{p(1)}{\left(1 + \frac{(k-p_2)^2+(l-p_3)^2}{p_4^2}\right)^{p_5}} \tag{7}$$

where first four parameters are same as in the case of Gauss function and p_5 is shape parameter.

2.3 PSF parameters optimization

In statistics, MLE is a method of estimating the parameters of a statistical model, Eq. (6). When applied to a data set and given statistical model, MLE provides estimates for the model's parameters. For a fixed set of data and certain statistical model, it produces a distribution that gives to the measured data the greatest probability, i.e., estimated parameters maximizes the likelihood function [6, 7].

When it is supposed that the data are Poisson distributed

$$\rho(x,\lambda) = \frac{\lambda^x}{x!}\mathrm{e}^{-\lambda} \tag{8}$$

then

$$\ln\rho = -\lambda + x\ln\lambda - \ln x!. \tag{9}$$

The opposite likelihood function can be written as

$$\phi = -\ln\mathcal{L} = \sum_{k=1}^{M}\sum_{l=1}^{N} -\ln\rho\big(\mathrm{x}(k,l), \mathrm{d}(k,l) + \mathrm{f}(k,l,\boldsymbol{p})\big) \to \min_{\boldsymbol{p}} \tag{10}$$

where $\mathrm{x}(k,l)$ is the analyzed light image, $\mathrm{d}(k,l)$ presents appropriate average dark frame and $\mathrm{f}(k,l,\boldsymbol{p})$ is the diffusion model, Eq. (6), whereof parameters are estimated.

Combination of Eq. (9) and (10) leads to the final form of function ϕ

$$\phi = c + \sum_{k=1}^{M}\sum_{l=1}^{N}\Big(-\mathrm{x}(k,l)\ln\big(\mathrm{d}(k,l) + \mathrm{f}(k,l,\boldsymbol{p})\big) + \mathrm{d}(k,l) + \mathrm{f}(k,l,\boldsymbol{p})\Big) \tag{11}$$

where c is some constant. The constant c is only data depending and can be set to satisfy $\phi \geq 0$ and obtain

$$\phi = \sum_{k=1}^{M}\sum_{l=1}^{N}\Big(-\mathrm{x}(k,l)\ln\big(\mathrm{d}(k,l)+\mathrm{f}(k,l,\boldsymbol{p})\big)+\mathrm{d}(k,l)+\mathrm{f}(k,l,\boldsymbol{p})+\mathrm{x}(k,l)\ln\mathrm{x}(k,l)-\mathrm{x}(k,l)\Big) \to \min_{\boldsymbol{p}}. \tag{12}$$

For purpose of minimization, MATLAB built-in function `fmincon` was applied.

3 Results

Analyzed astronomical objects were classified into three classes based on the bit depth of the analyzed image. Processed data were acquired in the 16 bit depth, thus the maximum intensity is 65 535. The interval of intensity values was uniformly divided into three classes, which can be written as follows:

- small object - maximum intensity in the analyzed area is less than 21 845,
- medium object - maximum intensity in the analyzed area is higher than 21 845 and less than 43 690,
- large object - maximum intensity in the analyzed area exceeds 43 690 and the top is given by the system resolution properties, thus 65 535.

Chosen objects that were used for an application of proposed methods can be seen in Fig. 1.

In Tabs. 1 and 2 are presented results of used models optimization. These tables contains information about $\phi_{\min}$, $\phi_{\max}$ values, mean and standard deviation of ϕ values for 50 evaluation of target function and estimated model parameters for ϕ_{min} value. Graphically are mean and standard deviation values shown in Fig. 5.

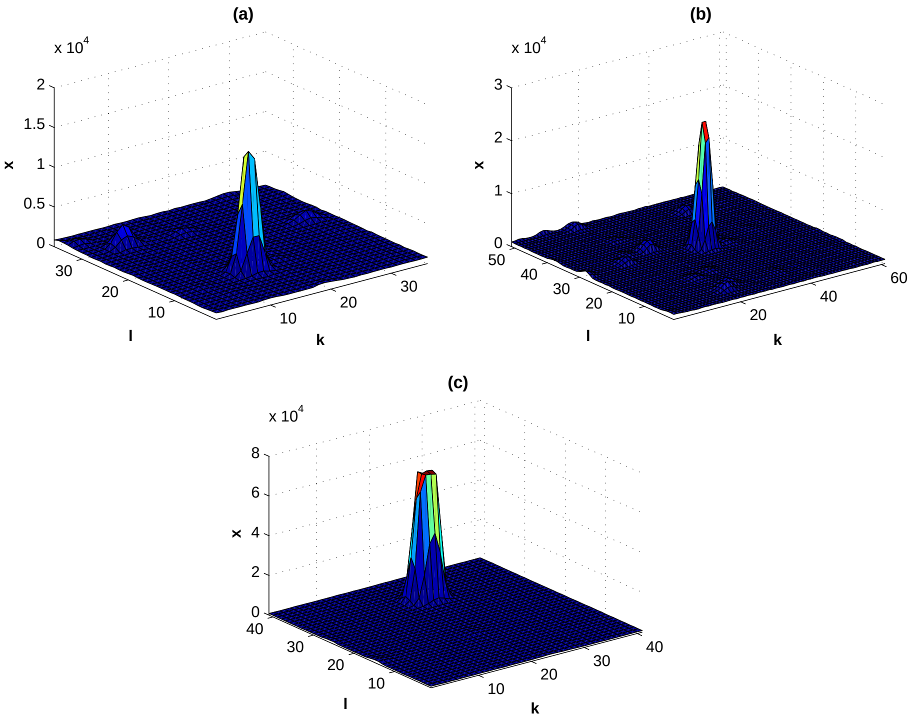

Figure 1: Objects, (a) Small, (b) Medium, (c) Large

Table 1: Optimization results of Gauss model

object	function values				optimum parameters			
	$\phi_{\min}$	$\phi_{\max}$	ϕ_{average}	ϕ_{std}	p_1	p_2	p_3	p_4
small	6.15×10^3	1.32×10^5	1.12×10^4	2.50×10^4	1.76×10^4	11.42	14.39	1.38
medium	1.10×10^4	3.80×10^5	7.74×10^4	1.43×10^5	2.75×10^4	26.94	33.44	1.69
large	5.53×10^4	9.55×10^5	1.63×10^5	2.95×10^5	6.95×10^4	31.71	23.52	1.97

Table 2: Optimization results of Moffat model

object	function values				optimum parameters				
	$\phi_{\min}$	$\phi_{\max}$	ϕ_{average}	ϕ_{std}	p_1	p_2	p_3	p_4	p_5
small	6.11×10^3	9.36×10^4	9.61×10^3	1.73×10^4	1.82×10^4	11.42	14.39	4.36	11.07
medium	1.11×10^4	2.48×10^5	1.58×10^4	3.35×10^4	2.78×10^4	26.94	33.44	10.00	36.34
large	5.83×10^4	2.90×10^5	7.69×10^4	6.35×10^4	6.79×10^4	31.71	23.51	10.00	26.49

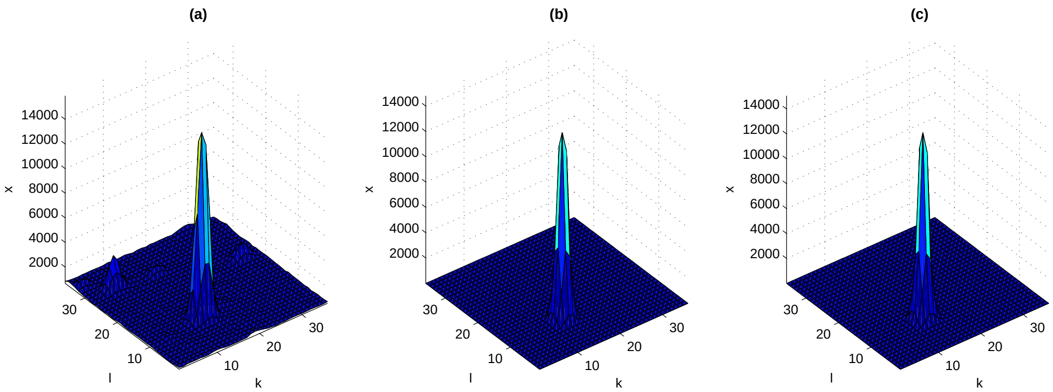

Figure 2: Graphical results of estimated models for $\phi_{\min}$ (a) Small astronomical object, (b) Estimated Gauss model, (c) Estimated Moffat model.

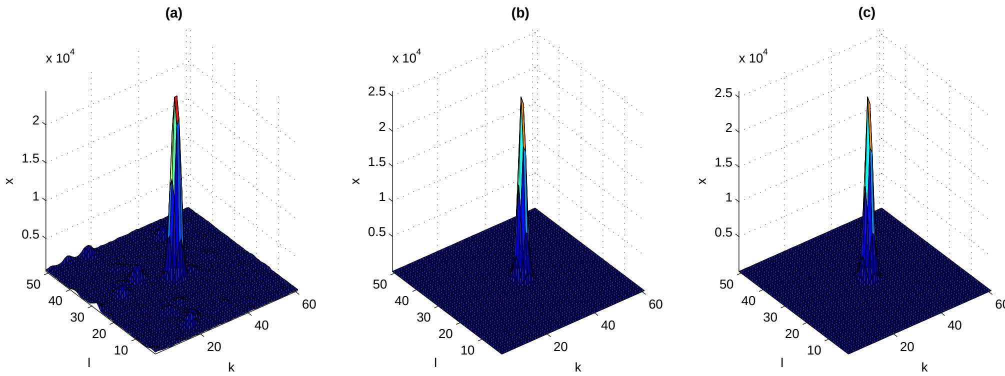

Figure 3: Graphical results of estimated models for $\phi_{\min}$ (a) Medium astronomical object, (b) Estimated Gauss model, (c) Estimated Moffat model.

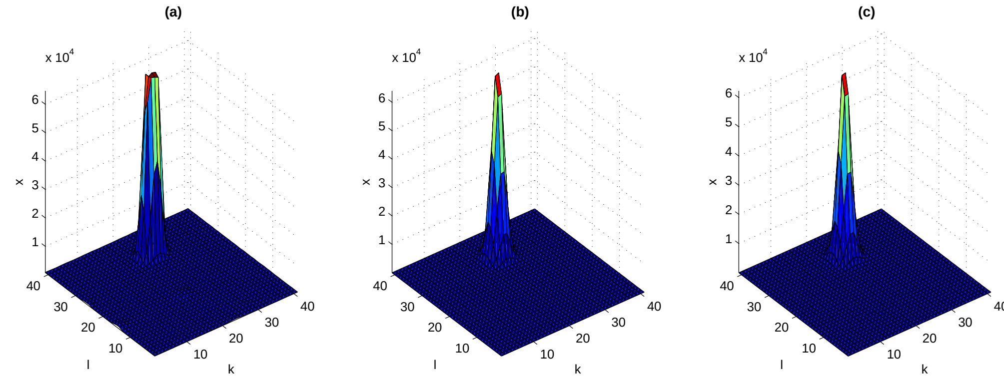

Figure 4: Graphical results of estimated models for $\phi_{\min}$ (a) Large astronomical object, (b) Estimated Gauss model, (c) Estimated Moffat model.

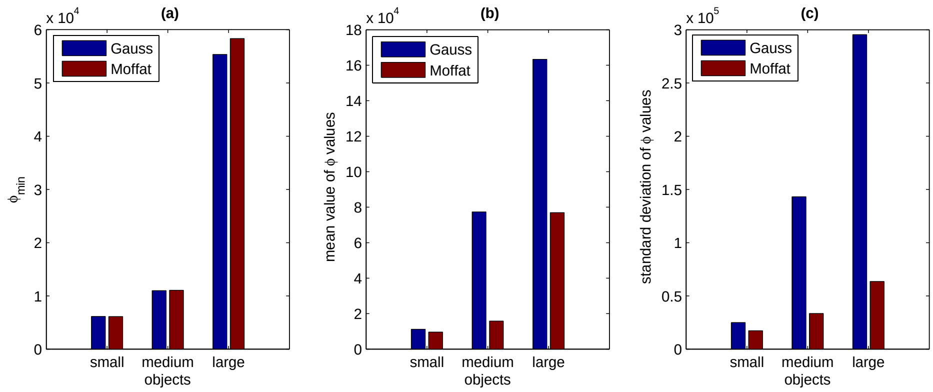

Figure 5: (a) $\phi_{\min}$ values for used models and analyzed objects, (b) Mean value of ϕ values for used models and analyzed objects and 50 evaluations, (c) Standard deviation of ϕ values for used models and analyzed objects and 50 evaluations.

4 Conclusion

This paper was devoted to processing of astronomical data and focused especially on detection, localization and modeling of astronomical objects. There were applied methods based on mathematical statistics, optimization methods and approaches used in astronomical photometry.

From presented results is obvious that Gauss model can be more suitable for modeling of astronomical objects, Fig. 5(a). When the mean values of ϕ and its standard deviaton are compared, Figs. 5(a) and (b), than the Moffat model gives us better results in case of more function evaluation. Thus, it is not easy to say which model is better and which is worse. It depends on exact application. These facts are the reason for further investigation of another more suitable models and different optimization algorithms.

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