MODELING OF THERMAL PROFILE INSIDE GUARDED HOT PLATE APPARATUS

A. Blahut, R. Strnad, L. Kňazovická

Czech Metrology Institute, Okružní 31, 638 00 Brno, Czech Republic

Abstract

Guarded hot plate apparatus (GHP) is a convenient tool to measure low thermal conductivities of insulating materials. Investigated material is placed between two plates (hot and cold) maintained at desired temperatures. As a consequence of temperature gradient, heat flow through the material occurs. Based upon Fourier law and knowledge of a power of hot plate heater coefficient of thermal conductivity can be determined.

Taking into account only conductive mechanism of heat transfer simplified model of temperature profile in GHP is presented in this paper. Model realized in MATLAB allows investigating the influence of many parameters (including geometry of GHP apparatus itself) on the desired temperature profile.

1 Introduction

Guarded hot plate method [1] is widely accepted method for determination of heat-transfer properties of thermo-insulating materials. The accurate values of thermal resistance of these materials are needed in many fields not only in the civil engineering. There is an effort to extend GHP measurements to a high temperature region keeping acceptable accuracy (expanded uncertainty of resulting values below 5 %). High temperatures represent higher demands for materials used. Also great attention is paid to own apparatus design to avoid undesirable heat losses [2].

The apparatus is designed to provide unidirectional heat flow from hot plate to cold plate through a specimen enabling the simple determination of desired heat transfer quantity (thermal conductivity, thermal resistance etc.). A scheme of usual double specimen type apparatus is shown in Figure 1, where particular layers of cylindrical shape are sandwich-like arranged.

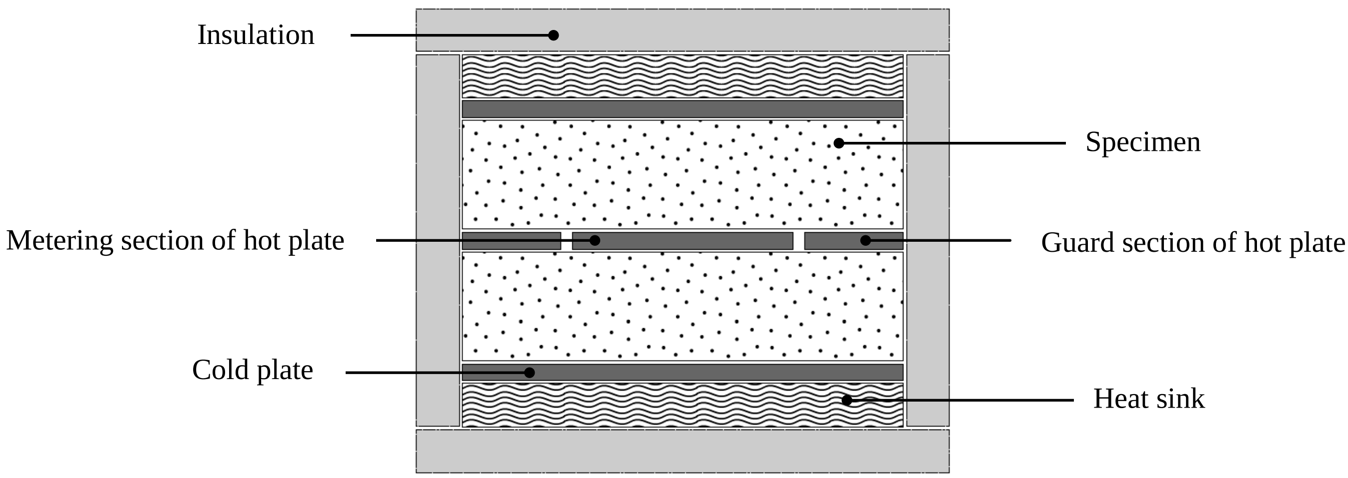


Figure 1 Scheme of double sided GHP apparatus

The goals of modeling temperature profile inside GHP apparatus are to confirm unidirectional character of heat flow in the metering section through the specimen and follow time evolution of the temperature profile estimating time needed to reach a steady state.

All calculations were performed using Matlab® R2010a.

2 Theoretical background

Usually heat transfer is categorized into three different mechanisms: conduction, convection and radiation. Modeling the temperature profile in this paper is limited only to the heat conduction which is described by Fourier's law:

$$\vec{\phi} = -\lambda \nabla T \tag{1}$$

where $\vec{\phi}$ is the heat rate per unit area (heat flux), λ coefficient of material's thermal conductivity and ∇T the temperature gradient. Assuming steady state and unidirectional heat flow between hot and cold plate of GHP apparatus (two specimen type), λ is calculated according to

$$\lambda = \frac{Q\delta}{2A\Delta T} \tag{2}$$

where Q stands for heater power of hot plate metering section, δ is specimen thickness, A metering section area and ΔT positive temperature difference between hot and cold plates. Number 2 corresponds to two-specimen arrangement of measurement.

To see time evolution of model system, following heat equation neglecting heat source is considered

$$\rho C_p \frac{\partial T}{\partial t} - \nabla \cdot (\lambda \nabla T) = 0\,. \tag{3}$$

In equation (3) ρ and C_p are density and specific heat capacity of specimen, respectively, and $\partial T / \partial t$ is time derivative of temperature. As a consequence of cylindrical symmetry of apparatus, the number of variable can be reduced by expressing (3) in the cylindrical coordinates

$$\rho C_p \frac{\partial T}{\partial t} - \frac{1}{r}\frac{\partial}{\partial t}\left(\lambda r \frac{\partial T}{\partial r}\right) - \frac{\partial}{\partial z}\left(\lambda \frac{\partial T}{\partial z}\right) = 0 \tag{4}$$

where r and z are radial and axial coordinates, respectively. In all presented model situations following Neumann boundary condition is assumed

$$\vec{n} \nabla T(0, z) = 0, \quad z_{\mathrm{min}} \le z \le z_{\mathrm{max}} \tag{5}$$

where $\vec{n}$ stands for unite normal vector aiming outwards.

3 Results

3.1 Edge guarding

On the edges of specimen heat losses usually occur leading to disruption of desired temperature profile. Four simple different situations of possible edge guarding were tested for *model specimen* of extremely low thermal conductivity coefficient (0.01 $Wm^{-1}K^{-1}$), thickness 0.1 m, density 500 kg m^{-3} and specific heat capacity 1000 J $kg^{-1}K^{-1}$. Particularly effect of 10 mm thick insulation (same properties as specimen except for thermal conductivity coefficient, λ =0.4 $Wm^{-1}K^{-1}$) adjacent to the specimen edge and different temperature edge profiles simulating additional edge heaters were examined. In figures 2 to 5 temperature profiles inside the specimen together with normalized vector field of heat flow are presented. Among four situations, as can be assumed, the edge guard using three heaters preserves best unidirectional heat flow in the metering section of hot plate.

1. No edge guarding, 820 °C on the specimen edge

$$T(r,0) = 840, \quad 0 \le r \le 0.152$$
$$T(r,0.1) = 800, \quad 0 \le r \le 0.152$$
$$T(0.152, z) = 820, \quad 0 \le z \le 0.1$$

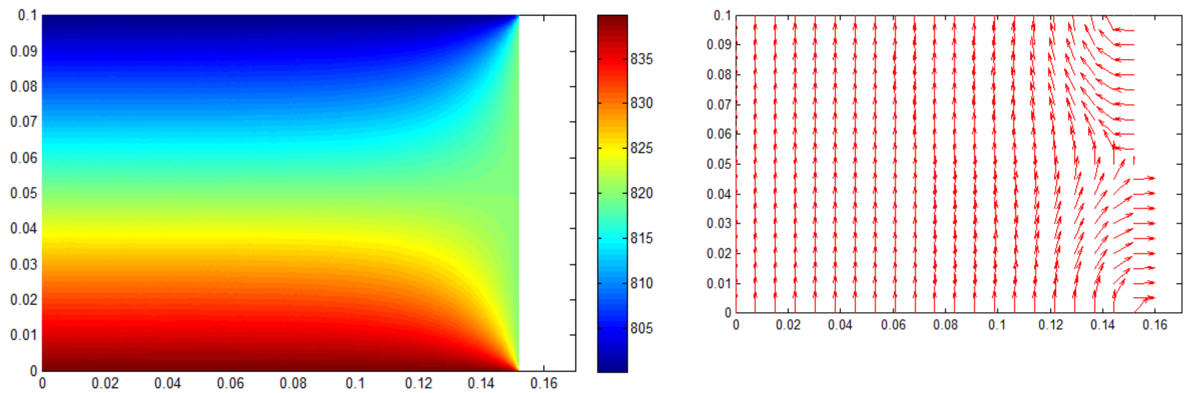


Figure 2 Temperature profile inside the specimen and normalized heat flow vector field, case 1

2. 10 mm insulation, one edge guard heater

$$T(r,0) = 840, \quad 0 \le r \le 0.162$$
$$T(r,0.1) = 800, \quad 0 \le r \le 0.162$$
$$T(0.162, z) = 820, \quad 0 \le z \le 0.1$$

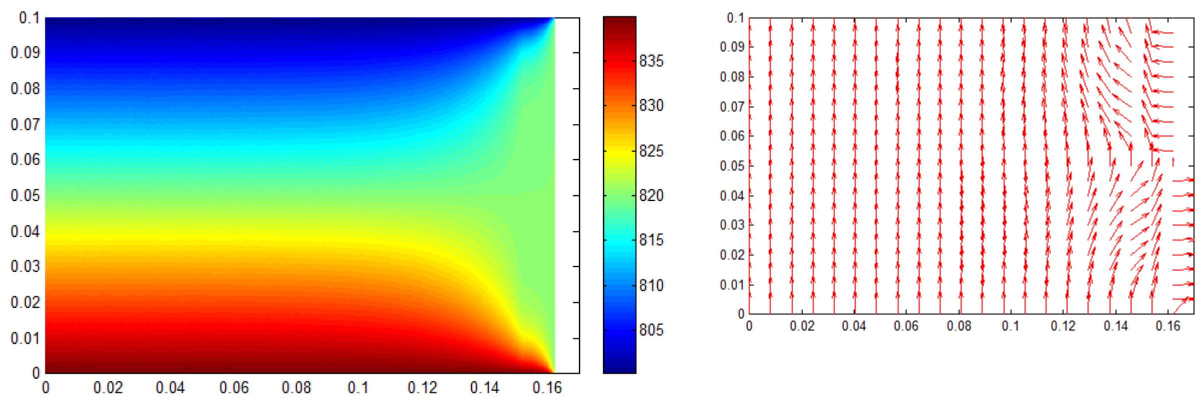


Figure 3 Temperature profile inside the specimen and normalized heat flow vector field, case 2

3. 10 mm insulation, two edge guard heaters

$$T(r,0)=840, \quad 0 \le r \le 0.162$$
$$T(r,0.1)=800, \quad 0 \le r \le 0.162$$
$$T(0.162,z)=840, \quad 0 \le z \le 0.5$$
$$T(0.162,z)=800, \quad 0.5 < z \le 0.1$$

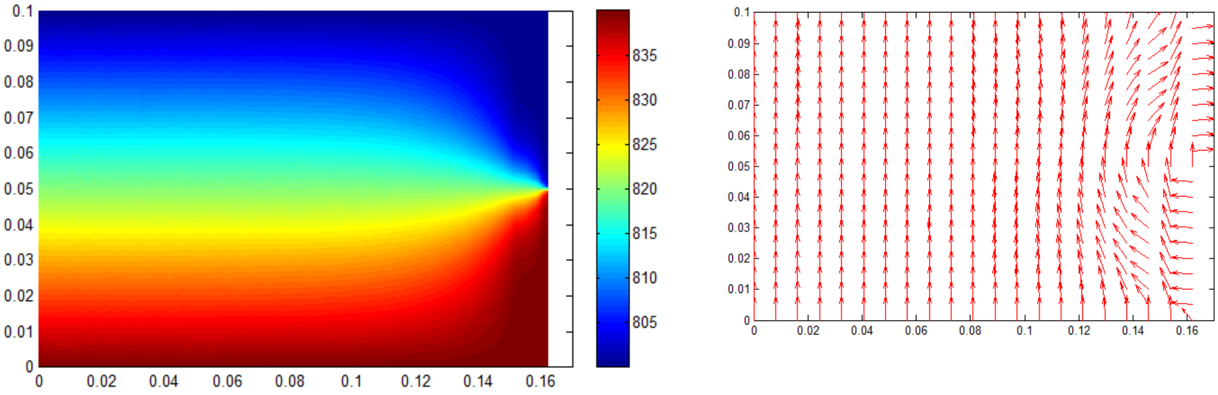


Figure 4 Temperature profile inside the specimen and normalized heat flow vector field, case 3

4. 10 mm insulation, three edge guard heaters

$$T(r,0)=840, \quad 0 \le r \le 0.162$$
$$T(r,0.1)=800, \quad 0 \le r \le 0.162$$
$$T(0.162,z)=840, \quad 0 \le z \le 0.1/3$$
$$T(0.162,z)=820, \quad 0.1/3 < z \le 0.2/3$$
$$T(0.162,z)=800, \quad 0.2/3 < z \le 0.1$$

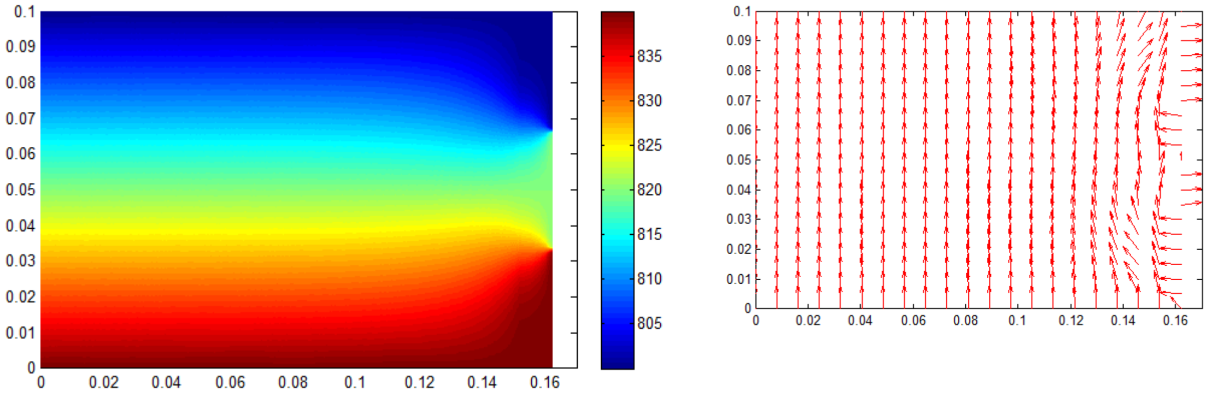
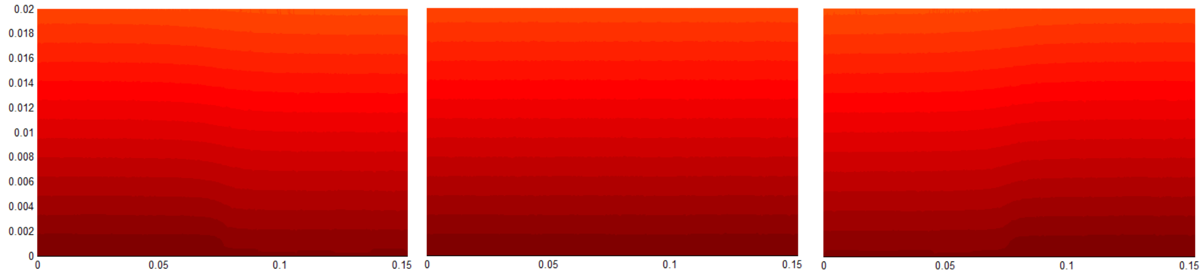


Figure 5 Temperature profile inside the specimen and normalized heat flow vector field, case 4

3.2 Gap imbalance

One of the factors which may influence unidirectional character of heat flow is gap imbalance – temperature difference between metering ($0 < r < 0.076$) and guard section ($0.076 < r < 0.152$) of hot plate. In figure 6 the effect of 0.5 °C imbalance on the temperature profile inside the *model specimen* is visualized.



$T(r,0) = 840.0, \quad 0 \le r \le 0.076$
$T(r,0) = 839.5, \quad 0.076 < r \le 0.152$

$T(r,0) = 840.0, \quad 0 \le r \le 0.152$

$T(r,0) = 840.0, \quad 0 \le r \le 0.076$
$T(r,0) = 840.5, \quad 0.076 < r \le 0.152$

Figure 6 Temperature profile inside specimen at various temperature differences between metering and guard section of hot plate

3.3 Time for reaching the steady state

Time dependence of temperature profile in the *model specimen* according to equation 3 was observed to roughly estimate time needed for reaching the steady state. Also influence of specimen thickness and coefficient of thermal conductivity on the stabilization rate was studied. For all cases 20 °C was set as an initial temperature of specimen. In figure 7 the dependence of stabilization time t(s) on heat conductivity coefficient in the semi-logarithmic coordinates at constant specimen thickness δ =0.1 m is illustrated. Figure 8 shows the influence of t(s) on specimen thickness at $\lambda = 1$. As can be seen, t(s) and λ are mutually inversely correlated, whereas the higher the specimen thickness keeping the same material properties, the higher the time to reach the steady state.

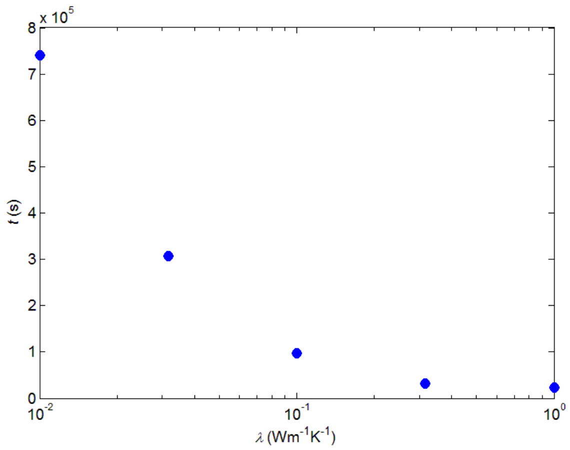


Figure 7 Stabilization time t vs. λ plot

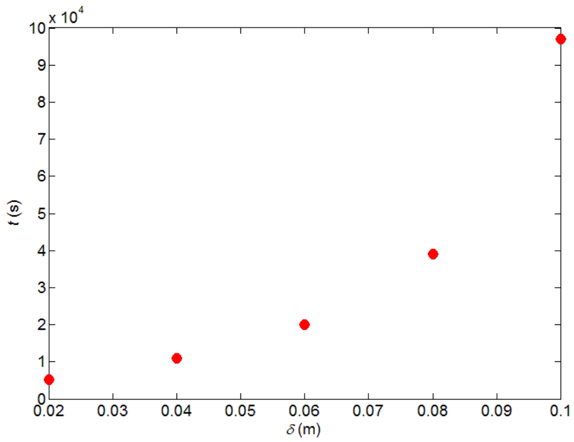


Figure 8 Stabilization time t vs. δ plot

4 Conclusion

Simple model of heat conduction was applied to cylindrical insulation specimen at conditions close to real high-temperature GHP apparatus. Particularly edge guarding, gap imbalance and time evolution of temperature profile inside specimen were studied. Although the model gives qualitatively reasonable results, it is quite far from accurate description of real GHP apparatus.

5 References

[1] Tye, R. P. Thermal Conductivity, *Nature* **204**, 636-637 (1964).

[2] Flynn, D. R.; Healy, W. M.; Zarr, R. R. High-Temperature Guarded Hot Plate Apparatus: Control of Edge Heat Loss. International Thermal Conductivity 28th Conference/International Thermal Expansion 16th Symposium. Proceedings. June 26-29, 2005, New Brunswick, Canada, DEStech Publications, Inc., Dinwiddie, R.; White, M. A.; McElroy, D. L., Editor(s)(s), 208-223 pp, 2005.

Aleš Blahut, Thermal Units Department – FM, Czech Metrology Institute, ablahut@cmi.cz

Radek Strnad, Thermal Units Department – FM, Czech Metrology Institute, rstrnad@cmi.cz

Lenka Kňazovická, Thermal Units Department – FM, Czech Metrology Institute, lknazovicka@cmi.cz