TECHNIQUES OF TIME-DOMAIN ANALYSIS
OF INTERCONNECTS

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Abstract

The paper deals with the Matlab application used for time-domain analysis of interconnect. For transient effect analysis two elementary implicit methods are used (Finite-Difference Time-Domain method and Wendroff method). The methods are very effective and deliver satisfactory results in case of linear lines with live conductors. For simulation is used time and spatial discretization of Maxwells equations in differential form. Results are presented in form of voltage and current distribution along a multiconductor transmission line. Typical situations are solved as an illustration of these methods.

1   Introduction

Transmission lines are used to transmit electrical signals between two or more points to distance hundreds or thousands of kilometers, for example, a source and a load. The term Multiconductor Transmission Line (MTL) typically refers to a set of *n + 1* parallel conductors. Very often in a technical practice MTLs are used, like as a feeder connecting radio transmitter with transmitting antenna, which is realized as a system of parallel conductors or as a coaxial line and in some case as a coaxial cable for smaller achievements. Other typical systems are conductive junctions in the midst of integrated circuits on folder modern processors, which work with high clock frequency. It is possible implement computer analyses single and complicated transmission lines in two ways. First is simulation in frequency domain, where are used to simulation harmonic signals about selected frequency. With this way it is possible analyze harmonic steady-state, wave length on lines, input impedance of lead or standing waves. The other way is simulation in time domain with using impulses by defined parameters and it is analyzed wave propagation along lead, reflection on the ends of leads, waveform distortion leading to losses or cross induction between near conductors. Surveyed transmission lines in this article are making by losses (resp. lossless) multiconductor transmission lines which are generally homogenous with frequency independent primary parameters and with zero initial conditions (current on inductors and voltage on capacitors).
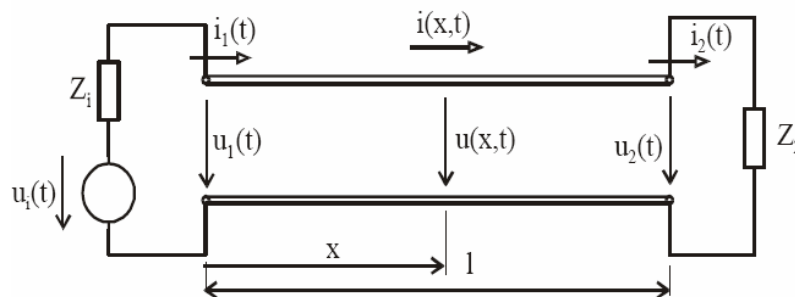

Figure 1: Schematic illustration of double conductor line

2   Mathematical model of transmission line

It is presumed single line realized as "live" conductors. The situation is illustrated in Fig. 1. On the left side of the figure, where usually existence of signal source so-called near end is supposed, on the other side there it is far end of lead.

At first, there are expressed conditions on elemental section of lead by length *dx*, which situation is illustrated on a substitution model in Fig. 2.
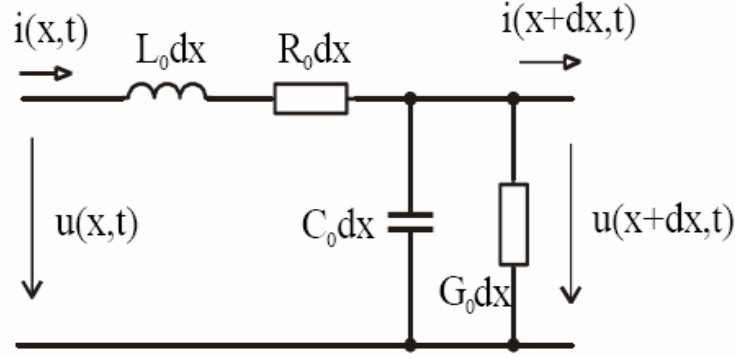
Figure 2: Elemental section substitution model of lead by length *dx*

A wave on a linear transmission line is described by partial differential equations for voltage *u (x, t)* and current *i (x, t)*. Both quantities are functions of time *t* and distance from near end of the line *x*. The equations are:

$$-\frac{\partial u(x,t)}{\partial x} = R_0 i(x,t) + L_0 \frac{\partial i(x,t)}{\partial t}, \tag{1}$$

$$-\frac{\partial i(x,t)}{\partial x} = G_0 u(x,t) + C_0 \frac{\partial u(x,t)}{\partial t}, \tag{2}$$

with corresponding initial (current and voltage distribution on line in time *t=0*) *u (x, 0)*, *i (x, 0)* and boundary conditions:

u (0, t) = u₁ (t), u (l, t) = u₂ (t),

i (0, t) = i₁ (t), i (l, t) = i₂ (t).

The symbols C_0[F/m], L_0[H/m], R_0[Ω/m]   and G_0[S/m] are primary line parameters, i.e. capacitance, inductance, resistance and conductance per unit length. In a general case, however, they may be functions of voltage or current [1].

3   Methods of analyze multiconductor transmission lines

3.1   FDTD methods

At last time FDTD (Finite-Difference Time-Domain) method is a popular computational technique for modeling. Two forms of these methods exist. First, explicit form, is the final difference method in time domain, where all of unknowns in surveyed time interval are calculated from well-known values from previous time interval. The method is not time and memory demanding, but it shows numerical instability and it is necessary to normalize parameters of line (primary parameters, length, time). Second form of FDTD methods is implicit form, where it is not necessary to normalize parameters, but the method requires calculation with inverse matrixes. The method is more numerical demanding. This paper deals only with implicit methods, concretely with FDTD and Wendroff method [2], [3].

3.2   Implicit FDTD method

For a numerical solution of the above described wave equation system (1) and (2), implicit FDTD method can be used. The line will be divided regularly with steps Δx, where $\Delta x = x_{k+1} - x_k$ and k marks spots of lead discretization ($k = 1, 2,..., N$). Time steps similarly are obtained from $\Delta t = t_l - t_{l-1}$, where l means spots of time discretization. Discretization in way x is possible presented instead a cascaded lead junction *(m+1)* Γ networks, with an inductor $L = L_0 \cdot d/m$ in series with resistor $R = R_0 \cdot d/m$ in longitude branches and with capacitor $C = C_0 \cdot d/m$ parallel with conductor $G=G_0 \cdot d/m$ in transverse branches by Fig. 2., where d is total length of lead. This will be the base of the FDTD method.

The basic equations (1) and (2) can be rearranged to get

$$u_k^{l+1} - u_{k-1}^{l+1} + \left(R_0 \Delta x + L_0 \frac{\Delta x}{\Delta t} \right)\left(i_k^{l+1} + i_{k-1}^{l+1} \right) = -u_k^l + u_{k-1}^l + L_0 \frac{\Delta x}{\Delta t} \left(i_k^l + i_{k-1}^l \right), \quad (3)$$

$$i_{k+1}^{l+1} - i_k^{l+1} + \left(G_0 \Delta x + C_0 \frac{\Delta x}{\Delta t} \right)\left(u_{k+1}^{l+1} + u_k^{l+1} \right) = -i_{k+1}^l + i_k^l + C_0 \frac{\Delta x}{\Delta t} \left(u_{k+1}^l + u_k^l \right), \quad (4)$$

with boundary conditions in form

$$u_1^{l+1} + \left(R_0 \Delta x + L_0 \frac{\Delta x}{\Delta t} \right) i_1^{l+1} = -u_1^l + L_0 \frac{\Delta x}{\Delta t} i_1^l + u_{i1}(n.\Delta t), \quad (5)$$

$$i_{m+1}^{l+1} - i_m^{l+1} + \left(G_0 \Delta x + C_0 \frac{\Delta x}{\Delta t} \right)\left(u_{m+1}^{l+1} + u_m^{l+1} \right) = -i_{m+1}^l + i_m^l + C_0 \frac{\Delta x}{\Delta t} \left(u_{m+1}^l + u_m^l \right) + u_{i2}(n.\Delta t) \ . \ (6)$$

To simulate the current and voltage distributions $u\,(t,\,x)$ and $i\,(t,\,x)$ along all lines of MTL in arbitrary time $t = n\Delta t$, the implicit formula can be expressed in a compact matrix form

$$\begin{bmatrix} u(x_1,...,x_{M+1}) \\ i(x_1,...,x_{M+1}) \end{bmatrix}^{l+1} = A^{-1}\left(B \begin{bmatrix} u(x_1,...,x_{M+1}) \\ i(x_1,...,x_{M+1}) \end{bmatrix}^l + D \right), \quad (7)$$

where coefficients of matrixes **A** and **B** are folded from four submatrixes with dimension $(m+1) \times (m+1)$ and are given by per-unit-length matrices, the matrix **D** is a column vector with dimension $2(m+1)$ is represented by signal sources and loads [3].

3.3 Wendroff method

This method has thus implicit character, generally coming out from same equations like FDTD method. Here again is used time and spatial discratization. Scheme of Wendroff formula is displayed in Fig. 3.
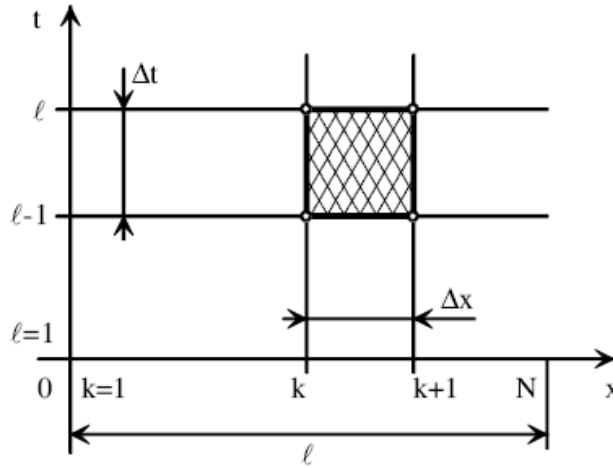

Figure 3: Scheme of Wendroff formula

For representation final difference scheme pursuant to basic wave Eq. (1) and (2) at points k and in time steps l we get

$$\frac{1}{2}\left(\frac{u_{k+1}^l - u_k^l}{\Delta x} + \frac{u_{k+1}^{l-1} - u_k^{l-1}}{\Delta x} \right) + \frac{R}{4}\left(i_{k+1}^l + i_k^l + i_{k+1}^{l-1} + i_k^{l-1} \right) + \frac{L}{2}\left(\frac{i_k^l - i_k^{l-1}}{\Delta t} + \frac{i_{k+1}^l - i_{k+1}^{l-1}}{\Delta t} \right) = 0, \quad (8)$$

$$\frac{1}{2}\left(\frac{i_{k+1}^l - i_k^l}{\Delta x} + \frac{i_{k+1}^{l-1} - i_k^{l-1}}{\Delta x} \right) + \frac{G}{4}\left(u_{k+1}^l + u_k^l + u_{k+1}^{l-1} + u_k^{l-1} \right) + \frac{C}{2}\left(\frac{u_k^l - u_k^{l-1}}{\Delta t} + \frac{u_{k+1}^l - u_{k+1}^{l-1}}{\Delta t} \right) = i_{0k}^l. \quad (9)$$

With application Eq. (8) and (9) on Eq. (1) and (2) and after representation boundary conditions

$x=0$ $F_1 = (u\ (0,\ t),\ i\ (0,\ t),\ t) = 0,$

$x=l$ $F_2 = (u\ (l\ ,t),\ i\ (l,\ t),\ t) = 0,$

and initial conditions

$u\ (x\ ,0) = 0,$

$i\ (x,\ 0) = 0,$

it is obtained a system of *2 x (N+1)* differential algebraic quadratics, which is possible script in matrix in form like in Eq. (7) [4].

4   Multiconductor transmission lines analyze in MATLAB

For functionality check of previous two methods (implicit FDTD and Wendroff method) there was created a program in MATLAB. There was created a GUI for the easier simulation, its example is in Fig. 4. With this application all parameters (primary parameters, sources, lead parameters etc.) are possible entered and start simulation.
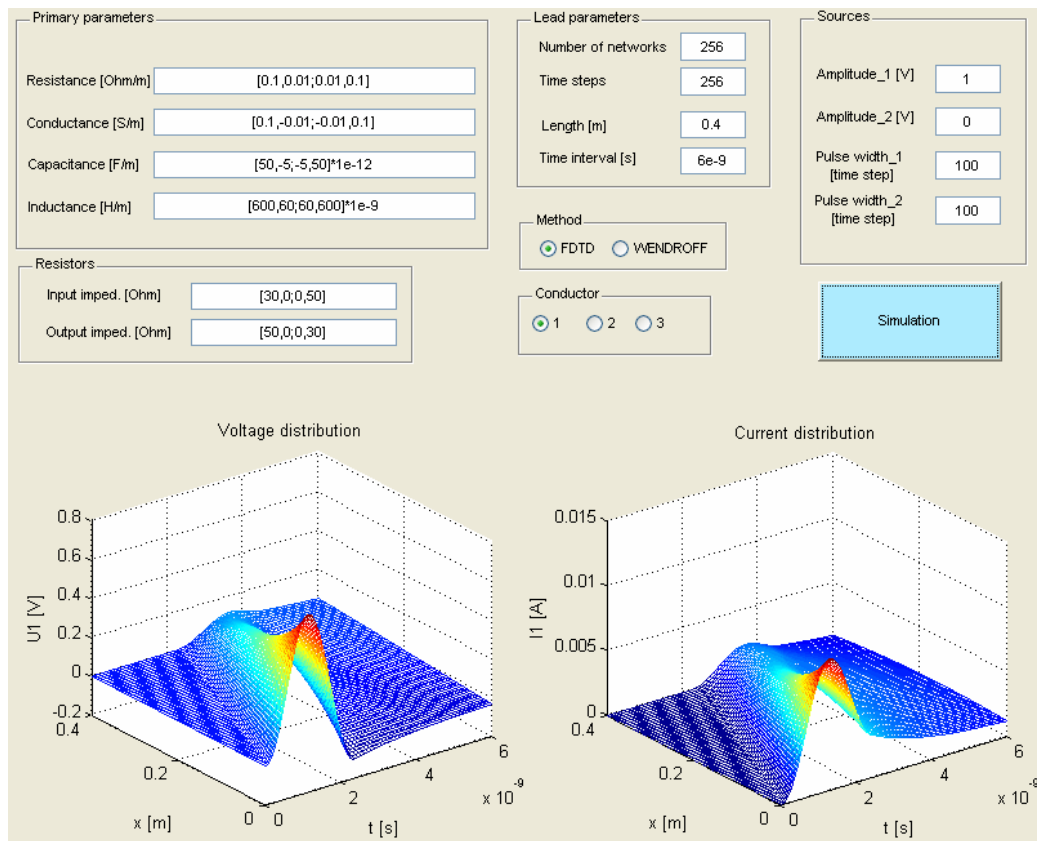

Figure 4: Voltage and current distribution on transmission line – GUI applied in Matlab

4.1   Matlab application description

In the first step it is necessary to order in matrix form primary parameters of transmission line. Matrix values are set in square brackets and values of even rows are divided by commas. The next row values are set after semicolons. All of these matrixes are symmetrical according to main diagonal and conductance and capacitance matrixes have expect main diagonal negative. Matrixes input and output resistors have a dimension N x N, where N means number of lines. Next is necessary to set length of lead in meters, time interval in seconds and number of Γ networks. Signal sources are presented with amplitude and pulse width. Last two options are choice of method and conductor number choice on which want to display voltage and current distribution.
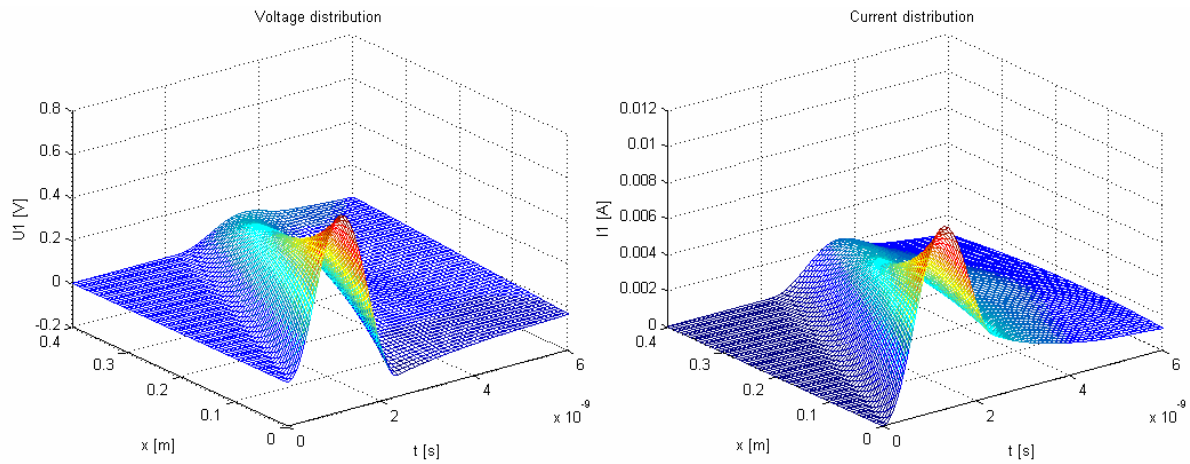
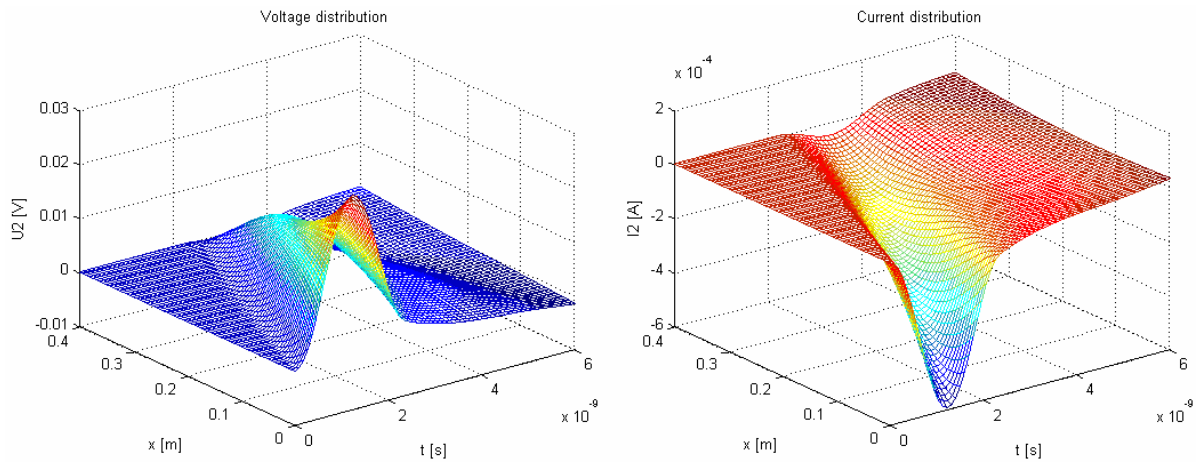Figure 5. Voltage and current distribution on 1st conductor – losses line

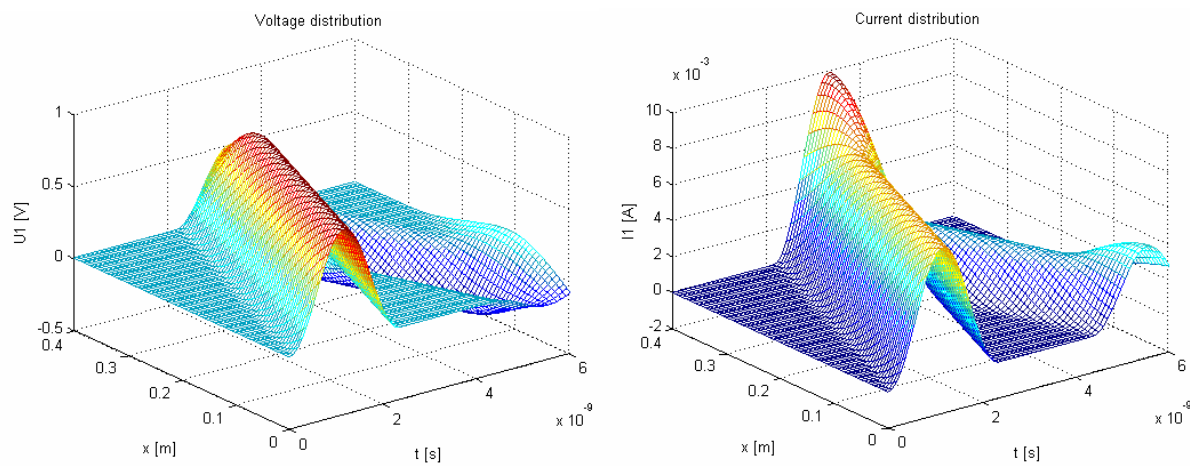Figure 6. Voltage and current distribution on 2nd conductor – losses line

Figure 7. Voltage and current distribution on 1st conductor – lossless line

After set-up of all parameters the simulation can be started. The results of case two-wire losses line simulation are shown in the Fig. 5, 6 and of case two-wire lossless line are represented in the Fig. 7, 8.
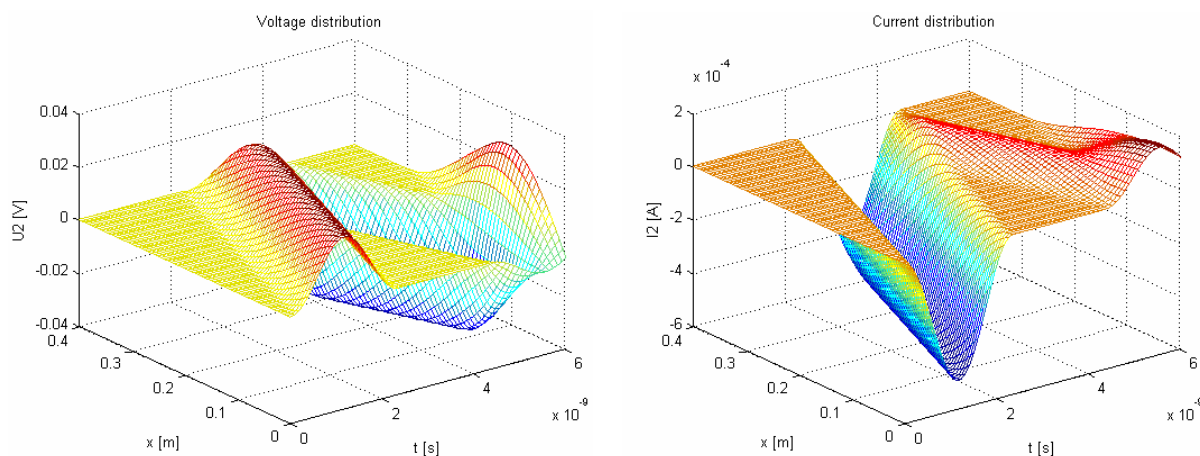
Figure 8. Voltage and current distribution on 2nd conductor – lossless line

5 Conclusion

In this paper there is presented possibility of an application of the FDTD methods to transient analysis on multiconductor transmission lines. The paper deals with possible ways of voltage and current wave distribution simulation along MTLs. First are presented basic characteristic of lines, their utilization in practice and there are formulated wave quadratics with boundary and initial conditions. Algorithms of numerical methods for transient effect solving in time domain are applied in Matlab. The application was demonstrated on suitable examples. Implicit variant of FDTD methods is a very stable numerical tool for the simulation of time-spatial dependences, it is not necessary with normalized primary parameters. Although the method is very time-consuming, by modifying input parameters of simulation it is possible formulate the exercise for simulation all sorts of situation.

Acknowledgement

References

[1] SEDLÁČEK, J., VALSA, J. *Elektrotechnika II.* VUTIUM, FEKT VUT v Brně, Brno, 2004.

[2] VALSA, J. *Simulace přechodných dějů na přenosových vedeních pomocí explicitní metody FDTD.* Sborník Semináře teorie obvodů STO-9, Brno, 2005.

[3] VALSA, J. *Simulace přechodných dějů na přenosových vedeních pomocí implicitní formy metody FDTD.* Sborník Semináře teorie obvodů STO-9, Brno, 2005.

[4] BENEŠOVÁ, Z., KOTLAN, V., MÜHLBACHER, J. *Surfe Waves propagation on Transmission Lines Induced by Lightning Stroke.* Power Systeme Conference & Exposition PSCE 2006. IEEE Power Engineering Society, Atlanta U.S.A., 2006.

[5] LÁBSKY, B. *Metody analýzy přenosových struktur v časové oblasti.* Diplomová práce. Ústav radioelektroniky, FEKT VUT v Brně. Brno, 2009.

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