COINTEGRATION MODELING OF INCOME TAX

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Abstract

The paper undertakes an estimation of values of natural person income tax (tax_t) from dependent work for following three years. It describes the influence of gross wage (w_t), employment ($empl_t$) and free working positions (fwp_t) on final tax revenue. Unit root tests are applied on logarithmic time series and after finding that all variables are first order cointegrated. Error correction models with exogenous variables approved to be the most accurate in sample and out of sample prognosis.

1 Methodology

The Johansen procedure provides a test statistic for determining *r*, the number of co-integrating relationships between the *n* variables in y_t as well as a set of *r* co-integrating vectors that can be used to construct error correction variables for the EC model.

$$\Delta y_t = \Gamma_1 \Delta y_{t-1} + \ldots + \Gamma_{k-1} \Delta y_{t-k+1} - \Psi y_{t-k} + \varepsilon_t \tag{1}$$

where $\Psi = \left(I_n - A_1 - A_2 - \ldots - A_k\right)$. If the matrix Ψ contains all zeros, (has rank=0), there are no co-integrating relationships between the variables in y_t. If Ψ is of full-rank, then we have *n* long-run equilibrium relationships, so all variables in the model are co-integrated. For cases where the matrix has rank $r < n$, we have *r* co-integrating relationships.

2 Data

The database contains quarterly time series from 1998 to 2007.[1] The year 2008 and 2009 was left for verification. Used values of tax income come from Main tax office, but they were recorded since first quarter 1998. Other data were provided by Statistical Office of Slovak republic. Nominal salaries are listed in common prices and other variables are in absolute value, for better lucidity, the coefficients are shown with three decimals and t-statistic values are rounded to two decimals. The values were logarithmically transformed before they enter the model.

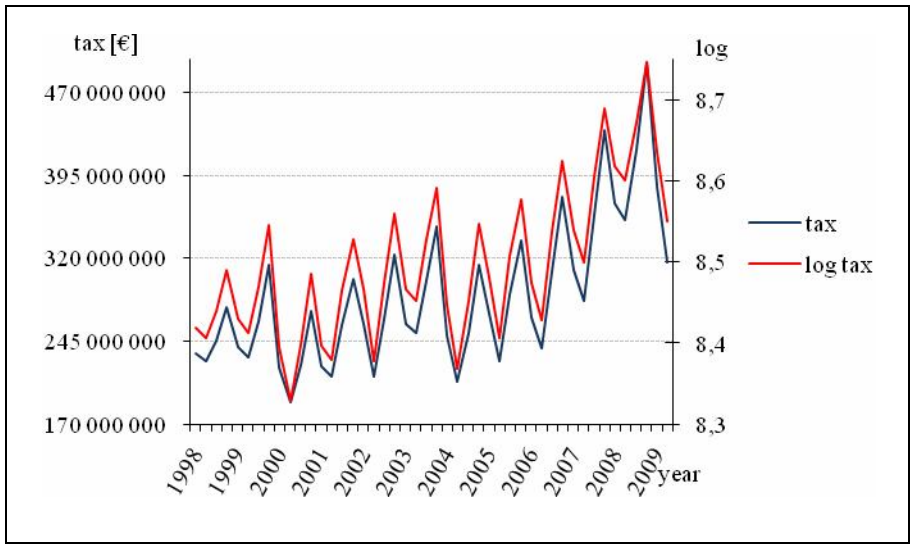

Figure 1: Tax and logaritmic time series of tax

[1] The absence of month values of registered employed number made necessary to choose quarterly data.

3 Designing the model

3.1 Unit root tests

The most famous test is the augmented Dickey-Fuller, another test is the Phillips-Perron. Both these tests use the existence of a unit root test as the null hypothesis. We reject the hypothesis of I(0), because our t-statistics for tax and gross wage are less than (in absolute value terms) the critical value at the 95 % level. Curves of employment and free working positions were tested analogically. Both curves contain the constant and we can claim, that this time series is first order integrated. By logarithmical transformation of variables the curves were created, which are first order integrated and there is long-time relation between them.

3.2 Johansen procedure

After the Johansen test (formula 1) determines the number of co-intergrating relationships, we can use these results along with the eigenvectors returned by the Johansen function, to form a set of error correction variables. These are constructed using y_{t-1} (the levels of y lagged one period) multiplied by the r eigenvectors associated with the co-integrating relationships to form r cointegrating variables. This is carried out by the ecm function, documented below.

3.3 EC model

The ambition was to choose model, which takes into account the most of appropriate econometric model attributes. One of possible suitable specifications is following form:

$$\Delta \log tax_t = -0.01 + 6{,}72\Delta \log empl_{t-1} + 0{,}78\Delta \log w_t - 0{,}43\Delta \log fwp_t - 0{,}59EC_{t-1} \qquad (2)$$

In the mentioned form of a model with correction member (EC = errorterm) the coefficient of short-time adaptation is important from statistical aspect, what confirms the legitimacy of using of correction mechanism. Parameter with value equal to -0,59 means fast convergence to long-time equilibrium, because the deviation from equilibrium is adjusted by 59 % back to equilibrium each quarter. The value (-0.59) is evaluated quarterly, i.e. taxes will return to equilibrium in half a year. That confirms hypothesis, that dynamics in tax system relations is very significant.

The model was created until fourth quarter 2007, year 2008 and 2009 was predicted and left for verification. Figure 2 shows values of EC model and the pseudoprognosis. Real values and predicted values using the EC model.

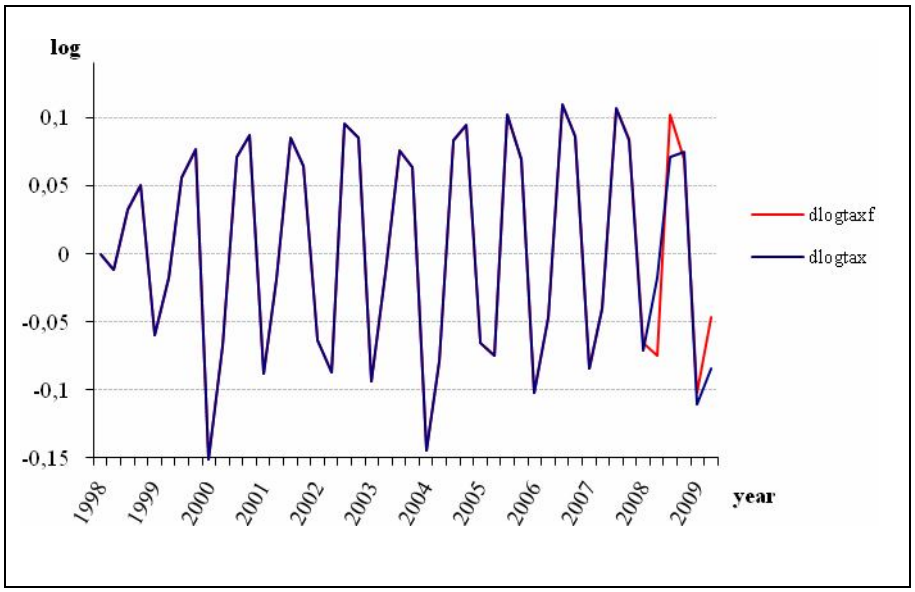

Figure 2: Pseudoprognosis

4 Model and prognosis

With EC model we calculated differences of logarithmically transformed values of tax income. By the use of exponential function on logarithmically transformed values and adding values of differences subtractions we get tax income values in money. Described model is relatively simple and gives good results. Correction coefficient is at interval from 89.8 to 103%.

It can be noticed that model is not suitable for prognosis, because there is no high autoregressive structure in described model and it was not possible to find another lags. Another problem was an insufficient database – it is not proper to make forecast for next three years from only ten years data. Results of this approach are shown in Figure 3. In this article I use the Econometrics toolbox for Matlab, copyrighted by James P.LeSage.

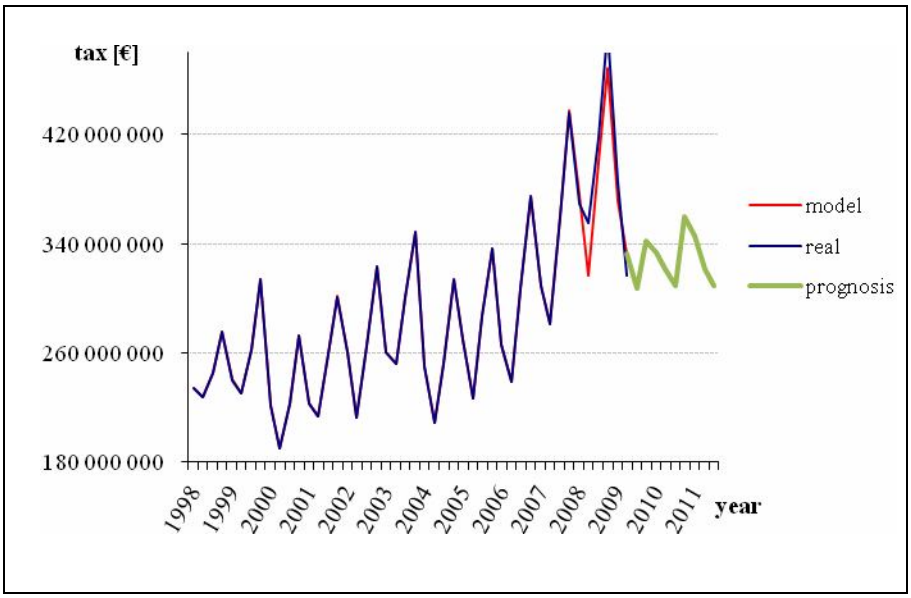

Figure 3: Software outputs

5 Conclusion

We have to say that making prognosis of tax income is not easy at all, because even relatively small deviations in percentual expression can affect result in a great measure.

The model is relatively accurate, because it comes out of real values and has small deviation in verification in 2009, on the other hand approach does not take the legislative into account and deviations from reality can be notable with time, especially in case when drafted model seems very good at the beginning and absence of significant variables becomes evident later.

We consider the detachment from economic theory as the main defect of model – it has only mathematic expression. Critics of this model claim distrust and congruently say that estimating behaviour of economy in equilibrium point is possible to predict only with taking economic theory into account. Anyway only the time can show accuracy of predictions and correctness of conclusions described above…

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