FAULT DETECTION AND ISOLATION BASED ON MARKOV CHAINS

J. Jirkovský

Department of Instrumentation and Control Engineering, Faculty of Mechanical Engineering, Czech Technical University

Abstract

There is described an approach in a Fault Detection and Isolation by means of Markov chains in this article. After first part which is devoted to terminology of the FDI there is a description of the Markov chains as a strong tool for dynamic systems modeling with extensions to the field of the FDI. There is also a simple practical example of the Markov chains FDI application for a two tank system with a PI controller.

1 Introduction

At the end of the 20th century and the beginning of the 21st century a technical development tends to more and more complex control systems which include more sophisticated algorithms. Therefore demands, such as reliability or safeness, are still increasing in importance. An early fault detection and isolation can help us to avoid totally breakdown of the system and an irreparable harm. In less hazardous cases the fault diagnosis can improve effectiveness of the system.

2 Terminology of fault diagnosis

A term fault has to be understood as unexpected change of the system although it needn't be represented as a physical damage or a breakdown [1]. Such faults obstruct or disturb normal function of an automatic system. It can cause unacceptable decrease of system performance or lead to critical situation. A monitoring system used for the fault detection and isolation is called a fault diagnosis system.

The fault diagnosis system is divided into two following parts [1]. The fault detection is a two-level decision making about a faulty state or a normal state of the system. The fault isolation is fault position finding such as which sensor or actuator is failed.

3 Description of dynamic systems by use of Markov chains

Complex real systems have some elements of stochastic behavior in most cases. Hence it is more suitable to describe these systems by the means of probability. Then the stochastic models can be used for the FDI. This method is included to the group of the process history based methods i.e. the probability model of the system is built from the large amount of the process data.

Markov chain is probability model which has so called Markov property. The Markov property means that probability of a state x_t is defined only by the last state x_{t-1} and not from the whole history of the stochastic process [2]. In terms of conditional probability it is

$$p(x_t \mid x_{t-1}) = p(x_t \mid x_{t-1}, x_{t-2}, \ldots, x_0). \tag{1}$$

The conditional probability $p(x_t|x_{t-1})$ is called transient probability and it is possible to create transient matrix where rows correspond to the particular states x_{t-1} and columns correspond to the particular states x_t. Each element of this matrix is transient probability $p_{ij}(t)$, i.e. it is the probability that the system move from the state i to the state j in the time t.

But there is more common variant of the Markov chains for the system description where the transient probability $p_{ij}(t)$ is used for description probability of an output $y_t = j$ when a regression vector is z_i. The regression vector consists of a combination of actual and past inputs and past outputs. Such a Markov chain describes a system with a discrete inputs and outputs, but it is possible to use it also for continuous systems with use of discretization. Hence the Markov chains are suitable for the description of general linear or nonlinear systems.

We will suppose the variant of Markov chain with the input vector $v_t = (v_t[1], v_t[2],..., v_t[\mu])$ where each discrete input $v_t[j] \in \varphi_v[j] = \{1, 2, \ldots, N_v[j]\}$ and only one discrete output $y_t \in \varphi_y = \{1, 2, \ldots, N_y\}$. The value of each input and output in given time is taken from the final set of values. This Markov chain is probability model which describes dependence of the discrete output y_t on the discrete regression vector z_t. This vector contains information about the final past history of inputs and output and the actual values of the inputs. It holds that

$$p(y_t \mid v_t, D^{t-1}) = p(y_t \mid z_t) \text{ for } t = t_0 + 1, t_0 + 2, \ldots, t_K \tag{2}$$

where $D_t = \{y_t, v_t\}$ and D^{t-1} is a set of D_t from the time 0 to the time t-1.

The first task is to determine a hypothesis ${}_iH$ ($i = 1,2,\ldots,r$) about structure of the Markov chain, i.e. to determine structure of the regression vector z_t. The second task is to find relevant Markov chain parameters ${}_iK$, i.e. to determine the transient matrix for the given hypothesis. We will also assume homogeneity of the Markov chain. The homogenous Markov chain has a time invariant transient matrix. General structure of the regression vector is

$$z_t = \left[v_t[1], v_{t-1}[1], \ldots, v_{t-k_{v1}}[1], v_t[2], \ldots, v_{t-k_{v\mu}}[\mu], y_{t-1}, \ldots, y_{t-k_y}\right]. \tag{3}$$

The particular vector ${}_iz_t$ by the hypothesis ${}_iH$ is given by determination of values $k_{v1},\ldots, k_{v\mu}$ and k_y in z_t and by arbitrary selection of it's elements. After next derivations and assumptions [3] we can obtain a final relation for probability of y_{t+1} without necessity of determination of ${}_iK$ parameters

$$p\left(y_{t+1} = \upsilon \mid {}_iz_{t+1} = {}_i\zeta, D^t, {}_iH\right) = \frac{n_{{}_i\zeta,\upsilon}(t)}{\sum_{\upsilon_p=1}^{N_y} n_{{}_i\zeta,\upsilon_p}(t)} \tag{4}$$

where $n_{{}_i\zeta,\upsilon}(t)$ is a number of event when $y_t = \upsilon$ and the regression vector ${}_iz_t = {}_i\zeta$. It is possible to understand this relation as a percentage occurrence of the event when the regression vector ${}_i\zeta$ is followed by the output with value $\tilde{\upsilon}$.

It is obvious from the previous relations that the only necessity for counting with Markov chains is the matrixes ${}_in(t)$ for $i = 1,2,\ldots,r$. These matrixes are called sufficient statistics. They can be counted by the recurrent relation

$${}_in_{{}_i\zeta,\upsilon}(t) = {}_in_{{}_i\zeta,\upsilon}(t-1) + \delta({}_i\zeta, {}_iz_\tau)\delta(\upsilon, y_\tau) \tag{5}$$

for $\upsilon \in \varphi_y, {}_i\zeta \in \varphi_{{}_iz}$ where Kronecker symbol δ is defined as $\delta(\alpha,\beta) = 1$ for $\alpha = \beta$ and $\delta(\alpha,\beta) = 0$ for $\alpha \neq \beta$.

For counting of the sufficient statistics can be used method of exponential forgetting for the systems with slow change of parameters [3].

4 Markov chains for FDI

A basic idea of using the Markov chains for FDI is given by the regression vector made from suitable system variables and the Markov chain output is the fault state of the system, so called a fault function. This discrete function gives a number which represent a type of the fault. For example the normal state of the system is denoted by number 1 and the individual faults are denoted by numbers 2,3, …, n_p+1. That means the system variable estimation isn't calculated there.

For the Markov chain identification is necessary to have sufficient amount of data from all expected system fault states and normal states. Then the sufficient statistics of the system is made from this data. It represents relations between the system variables and the system faults.

A basic method for the FDI by the use of Markov chains consist of a learning stage and a following diagnosis stage.

The learning stage is the first. The Markov chain with a chosen regression vector is learned by the data from the monitored system. For good performance of the following diagnosis is necessary to use the data which describes all of the normal and fault system states. The value of the fault function is assigned to the each regression vector made from the data which has been collected in appropriate fault states. The sufficient statistics is made from these marked data by Eq. (5) and it describes monitored system from the FDI point of view.

When the learning stage is completed the FDI system is switched to the diagnosis mode. The regression vector is made form the on-line measured system data. The output of the FDI is a probability distribution of all previously identified system faults and the normal state by Eq. (4). This information is then given to an operator or a supervision system.

The next method is a real-time fault diagnosis with supervised training [4]. This approach can be used without previously collected system data directly with the running system. The Markov chain is learned by orders of the supervisor simultaneously with the fault diagnosis by use of actual knowledge about the system faults.

However the difference between these approaches is mostly in implementation of a user interface. The Markov chain is used in the same way for the both methods. It is also possible to combine these ones.

The main advantage of the FDI by use of the Markov chains is easy using with linear as well as nonlinear systems. The main drawback is a large volume of the sufficient statistics for complex systems.

5 Illustrative example

A two tank system with a PI controller is chosen as an example of the FDI by use of the Markov chains Fig. 1. A controlled value is a water level in the second tank. It is controlled by the PI controller by the first (input) valve q_1.

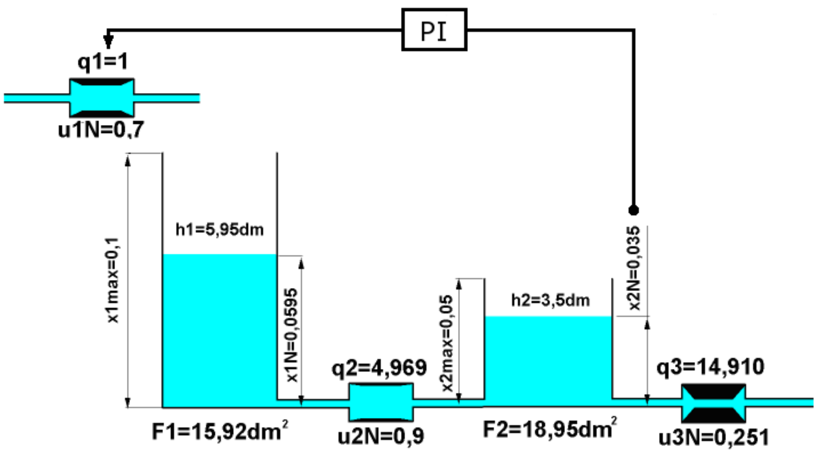

Figure 1: Two tank system with PI controller

The values of the system parameters are in relative form. For example the water levels x_1 and x_2 are related to the water level in a water source h_0=10m, etc.

The FDI algorithm was built by the principles of learning and following diagnosis method. In the learning stage the number of occurrence of the given regression vector is put to the sufficient statistics. The rows denote values of the regression vectors and the columns denote values of the fault function. If the algorithm finds a new regression vector, one row is added to the sufficient statistics matrix and a number one is put to the appropriate column. Then the transfer matrix between the regression vector and the fault variable is computed.

In the diagnosis stage the actual regression vector is compared with the vectors in the sufficient statistics. An output of the FDI is the row of the transfer matrix corresponding with the vector which is closest to the actual vector. The probability distribution of the fault function is given by this row.

Very important for the proper function of the algorithm is a choice of the regression vector. There are chosen the discretized values of the water level in the first tank, the water level in the second tank and the actuator output in our example. These values are discretized and normalized by a linear division of their ranges to 100 values from 0 to 99. The algorithm counting period is T=0.1s. The form of regression vector is

$$z(k) = [x_1(k), x_2(k), u_1(k), x_1(k-4), x_2(k-4), u_1(k-4), x_1(k-9), x_2(k-9), u_1(k-9)]. \qquad (6)$$

There are chosen two fault states. The first fault is decrease of the second valve opening from the value u_2=0.9 to the value u_2=0.75 and the second fault decrease of the second valve opening from the value u_3=0.251 to the value u_3=0.2. The PI controller is designed to work well in the case of occurrence of these faults. The set point of the second tank water level is changed in the interval $w_{x2}=\langle 0.03, 0.04\rangle$. It is necessary for the FDI to be able to recognize the fault state for the different set points.

The learning stage was performed at first. The fault variables are f_p=1 for normal state, f_p=2 for the decrease of the second valve opening and f_p=3 for the decrease of the third valve opening. Each state of the system was learned for the time period 1200s with the random change of the set-point with period 200s. After the learning the diagnosis was performed. There were repeated the three system states with the period of change 600s for each state with the random change of the set-point with period 200s. The diagnosis stage is depicted on (Fig. 2, Fig. 3, Fig. 4 and Fig. 5).

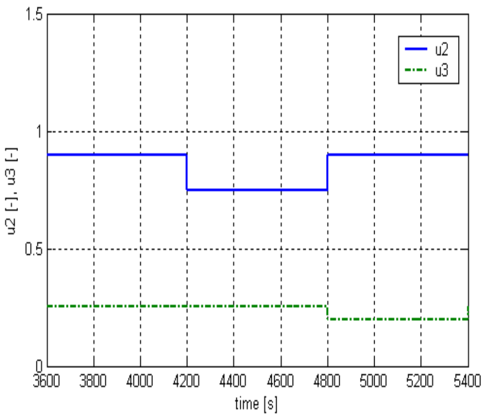

Figure 2: Valves opening during the diagnosis

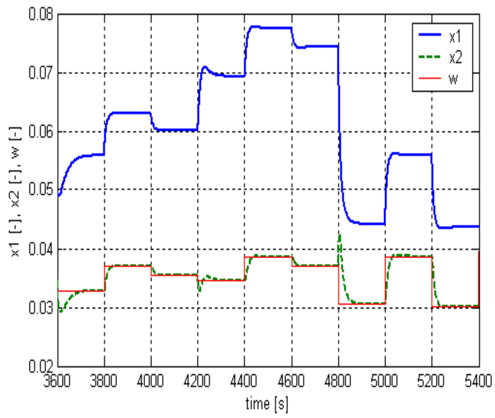

Figure 3: Water levels and the set point during the diagnosis

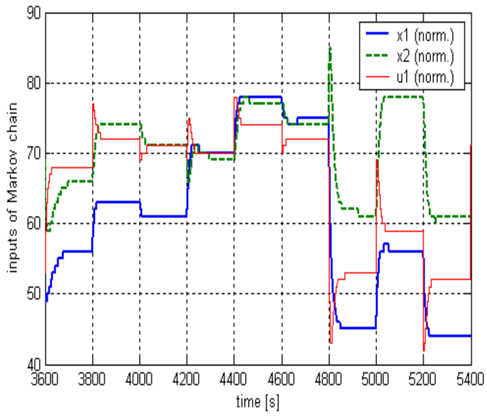

Figure 4: Discretized and normalized system variables during the diagnosis

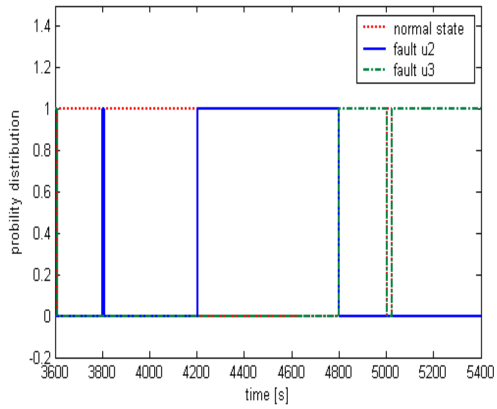

Figure 5: Probability distribution of the fault function (output of the FDI) during the diagnosis

It is obvious from the Fig. 5 that the FDI algorithm performed the fault diagnosis very well. Only in two short time sections about 3800s and 5000s the state was wrongly recognized. But it is possible to replace this event for example with using of a logical filter to some minimal time interval of the fault state before an alarm is used.

6 Conclusion

The FDI is a still expanding branch of automatic control which is used not only for special systems but also for common appliances in these days. There was described the stochastic approach to the systems in this article. The using of the Markov chains is a good and simple method for the FDI and it is possible to use them for linear as well as nonlinear systems. The more extensive information about this approach can be found in [4] and [5].

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