MODELING OF A FLUID-INDUCED ROTOR INSTABILITY

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Abstract

The paper deals with simulation of the rotor vibration in a journal bearing. The rotor is maintained in equilibrium during rotation by a fluid pressure wedge created by an oil film. These bearing forces can be modeled as a rotating spring and damper system. The rotor motion is described by complex coordinates of the rotor centerline. The real part of the position vector is a rotor displacement in the X-direction while the imaginary part is a displacement in the perpendicular direction, as we say in the Y-direction. The equation of motion is containing complex variables as a function of time. Equation parameters are complex quantities as well. The paper demonstrates how to solve such an equation using Simuling.

1 Introduction

There are many ways how to model a rotor system, but this paper prefers an aproach, which is based on the concept developed by Muszynska [1] supported by Bently Rotor Dynamics Research Corporation [2]. The reason for this is that this concept offers an effective way to understanding the rotor instability problem and excitation a vibration mode at the frequency, which is a fraction of the rotor rotational frequency. However, this simple mathematical model is inapplicable for solving practical technological problems. Another approach can be based on the lubricant flow prediction using a FE method for Reynolds equation solution, see [6] for instance. These more sophisticated methods do not allow analyzing the dynamic system stability.

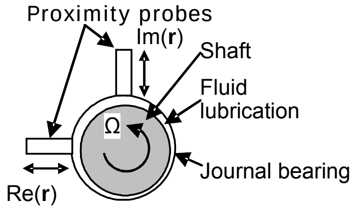

Figure 1: Journal coordinates

The arrangement of proximity probes in a rotor system is shown in figure 1. Let the rotor angular velocity is designated by Ω. It is assumed that the sleeve of the journal bearing is fixed while rotor is rotating at the mentioned angular velocity. This paper proposes to use complex variables to describe motion of the rotor in the plane, which is perpendicular to the rotor axis. The position of the journal centre in the complex plane, which origin is situated in the bearing centre, is designated by a position vector **r**.

2 Lumped parameter model of the rotor system

The internal spring, damping and tangential forces are acting on the rotor. The external forces refer to forces that are applied to the rotor, such as unbalance, impacts and preloads in the form of constant radial forces. All these external forces are considered as an input for the mathematical model based on the mentioned concept [4].

The fluid pressure wedge is the actual source of the fluid film stiffness in a journal bearing and maintains the rotor in equilibrium. As Muszynska has stated these bearing forces can be modeled as a rotating spring and damper system at the angular velocity $\lambda\Omega$ (see figure 2), where λ is a parameter, which is slightly less than 0.5. The parameter λ is denominated by Muszynska [1] as the fluid averaged circumferential velocity ratio. It is assumed that the rotating journal drags the fluid in a space between two cylinders into motion and acts as a pump. It is easy to understand that the fluid circular velocity is varying across the gap as a consequence of the fluid viscosity: At the surface of the journal, the fluid circular velocity is the same as the journal circumferential velocity and at the surface of the of the bearing, the fluid circular velocity is zero. The angular velocity $\lambda\Omega$ can be considered as the average angular velocity of the fluid but this quantity is only a fictive value. In fact, the angular velocity of the mentioned spring and damper system can be determined.

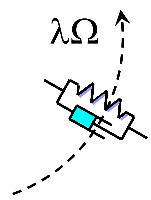

Figure 2: Model of oil film

The validity of Muszynska's assumption can be verified by an experiment. It is known that an oscillation starts when the rotor RPM crosses up some value and stops when RPM crosses down the other one. Some very sophisticated experiment shows that when the rotor system is excited by a non-synchronous perturbation force with respect to the rotor rotational speed the resonance appears at the frequency, which is approximately equal to $\lambda\Omega$. The simulation is prepared to prove the same properties of the mathematical model, which is based on Muszynska's teorie.

Fluid forces acting on the rotor in coordinates rotating at the same angular frequency as the spring and damper system are given by the formula

$$\mathbf{F}_{rot} = K\,\mathbf{r}_{rot} + D\dot{\mathbf{r}}_{rot}, \tag{1}$$

where the parameters, K and D, are specifying proportionality of stiffness and damping to the journal centre displacement vector $\mathbf{r}_{rot}$ and velocity vector $\dot{\mathbf{r}}_{rot}$, respectively. The spring force acts opposite to the displacement vector. Assuming constant values of K and D (isotropic rotor system) and independence of these parameters on the journal eccentricity, the system is considered to be linear.

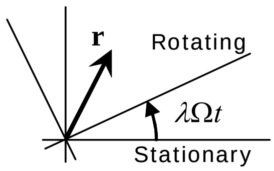

Figure 3: Transform to stationary coordinates

To model the rotor system, the fluid forces have to be expressed in the stationary coordinate system, in which the journal centre displacement and velocity vectors are designated by $\mathbf{r}$ and $\dot{\mathbf{r}}$, respectively. Conversion of the complex rotating vector $\mathbf{r}_{rot}$ to the stationary coordinate system can be done by multiplication of this vector by $\exp(j\lambda\Omega\,t)$, which is the same as multiplying the vector in the stationary coordinates by $\exp(-j\lambda\Omega\,t)$, see figure 3. The relationship between the mentioned vectors in rotating and stationary coordinates are given by the formulas

$$\begin{aligned}\mathbf{r}_{rot} &= \mathbf{r}\exp(-j\lambda\Omega\,t)\\ \dot{\mathbf{r}}_{rot} &= (\dot{\mathbf{r}} - j\lambda\Omega\,\mathbf{r})\exp(-j\lambda\Omega\,t)\end{aligned} \tag{2}$$

Substitution into the fluid force equation results in the following formula

$$\mathbf{F}_{rot} = K\,\mathbf{r}_{rot} + D\dot{\mathbf{r}}_{rot}, \tag{3}$$

where the complex term $jD\lambda\Omega\,\mathbf{r}$ has the meaning of the force acting in the perpendicular direction to the vector $\mathbf{r}$ and this force is called tangential. As the rotor angular velocity increases, this force can become very strong and can cause instability of the rotor behavior.

As it was mentioned the rotor is under influence of the perturbation external forces, for instance produced by unbalance mass or simply by gravity. This external perturbation force is assumed to be rotating at the angular velocity ω, which is considered to be completely independent on the rotor angular velocity Ω to obtain general solution [1,2]. The rotor is perturbed by the non-synchronous force with respect to the rotor angular frequency. The unbalance force, which is produced by unbalance mass m mounted at a radius r_u and rotating at the angular velocity ω, acts in the radial direction and has a phase δ at time $t = 0$

$$\mathbf{F}_{Perturbation} = mr_u\omega^2\exp(j(\omega t + \delta)), \tag{4}$$

The equation of motion for a rigid rotor rotating at the steady-state rotation speed and operating in a small, localized region in the journal bearing is as follows

$$M\ddot{\mathbf{r}} = -K\,\mathbf{r} - D\dot{\mathbf{r}} + jD\lambda\Omega\,\mathbf{r} + mr_u\omega^2\exp(j(\omega t + \delta)), \tag{5}$$

where M is the total rotor mass. After rearranging the ordinary linear differential equation (5), the equation of motion with constant coefficients is obtained

$$M\ddot{\mathbf{r}} + D\dot{\mathbf{r}} + (K - jD\lambda\Omega)\,\mathbf{r} = mr_u\omega^2\exp(j(\omega t + \delta)), \tag{6}$$

The position vector $\mathbf{r}$ describes a motion in the plane, which is perpendicular to the rotor axis. The trajectory of the rotor centerline is called an orbit.

3 Equation of motion as a servomechanism

The rotor/fluid wedge bearing/system can be demonstrated as a servomechanism working in the closed loop, which is shown in figure 4. The direct and quadrature dynamic stiffness is introduced according to the acting force direction.

$$\begin{aligned} K_{Direct}(j\omega) &= K + j\omega D - M\omega^2 \\ K_{Quadrature}(j\omega) &= -j\lambda\Omega D \end{aligned} \tag{7}$$

The individual transfer function $1/K_{Direct}(j\omega)$ (direct dynamic compliance) is stable. The feedback path in the closed-loop system acts as a positive feedback and introduces instability for the closed-loop system. The gain of the positive feedback depends on the angular velocity Ω. The closed-loop system is stable for the low rotor rotational speed. There is a margin for the stable behavior. If the gain of the positive feedback crosses over a limit value then the closed-loop becomes unstable. The properties of the unstable behavior can be analyzed using the servomechanism in Figure 4.

The stability of the closed-loop dynamic system is depending on the open-loop frequency transfer function for $s = j\omega$

$$G_0(j\omega) = \frac{K_{Quadrature}(j\omega)}{K_{Direct}(j\omega)} = \frac{-\lambda\Omega D}{\omega D - j\left(K - M\omega^2\right)}, \tag{8}$$

As it is known the closed-loop dynamic system is stable according to the Nyquist stability criterion if, and only if, the locus of the $G_0(j\omega)$ function in the complex plane does not enclose the (-1,0) point as ω is varied from zero to infinity [7]. Enclosing the (-1,0) point is interpreted as passing to the left of the mentioned point. When the steady-state vibration occurs, the stability margin is achieved. The locus of the $G_0(j\omega)$ function, describing the steady-state vibration, meets the (-1,0) point, therefore

$$G_0(j\omega_{Crit}) = -1. \tag{9}$$

An angular frequency, at which a system can oscillate without damping, is designated by ω_{Crit}. Substitution (9) into (8) results in formulas for the oscillating frequency

$$\omega_{Crit}^2 = K/M \quad \text{and} \quad \omega_{Crit} = \lambda\Omega. \tag{10}$$

It can be concluded that the frequency of the rotor subharmonic oscillation is the same as the fluid average angular velocity. The measurement shows that the value of the parameter λ is equal to 0.475. This result confirms the introductory assumption about the fluid forces acting on the rotor. The stability margin corresponds to the mechanical resonances of the rigid rotor mass supported by the oil spring. It can be noted that the frequency ω_{Crit} is not equal to the rotor critical speed when the vibration is excited by the rotor unbalance.

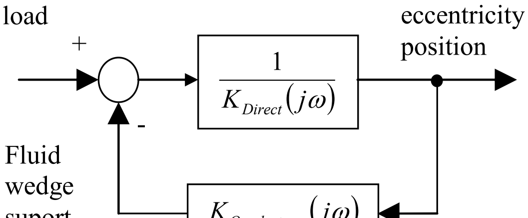

Figure 4: Shaft/fluid wedge bearing/system as a servomechanism

If the system were linear, then the unstable rotor vibration would spiral out to infinity when the rotor angular frequency crosses the so-called Bently-Muszynska threshold

$$\Omega_{Crit} = 2\pi f_{Crit} = \frac{\sqrt{K/M}}{\lambda}. \tag{11}$$

4 Simulink model of the rotor system

The equation of motion (6) contains as an unknown function of time a complex vector $\mathbf{r}(t)$ and the equation parameters are complex quantities as well. The complex function can be replaced by the real and imaginary functions and solved as many similar models. In this paper it is preferred an approach based on the Simuling feature allowing to connect blocks by a complex signals. Except of the integration function, all the blocks employed in the Simuling model for the motion equation (6) can work with the complex parameters and functions. The integration subsystem is shown in figure 5. The complex signal is decomposed into the real and imaginary parts for individual integration operation and then they are combined to the complex signal again.

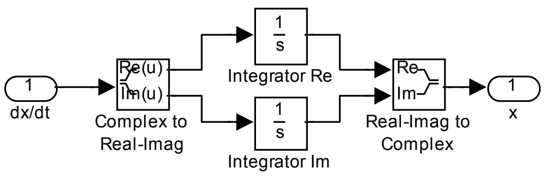

Figure 5: Integration subsystem

The Simulink block diagram for the motion equation is shown in figure 6. The system is excited by an unbalance force rotating at the same angular velocity Ω (OMEGA) as the rotor and by the non-synchronous perturbation force rotating by the angular velocity ω (omega), which amplitude is proportional to the square of the angular velocity.

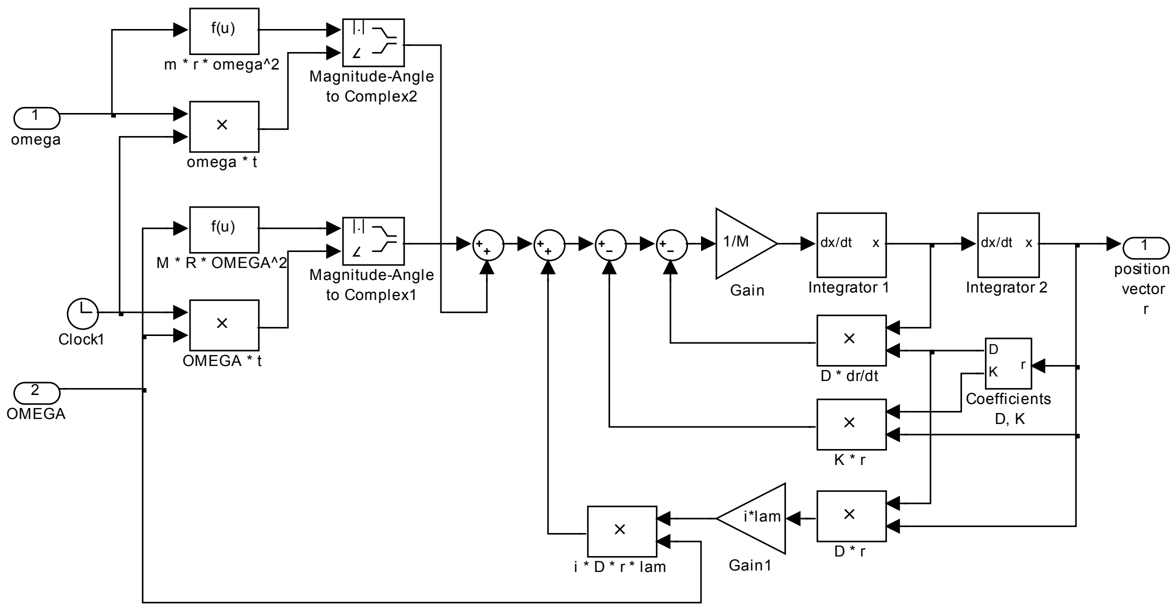

Figure 6: Model of a journal motion in a plane perpendicular to the rotor axis

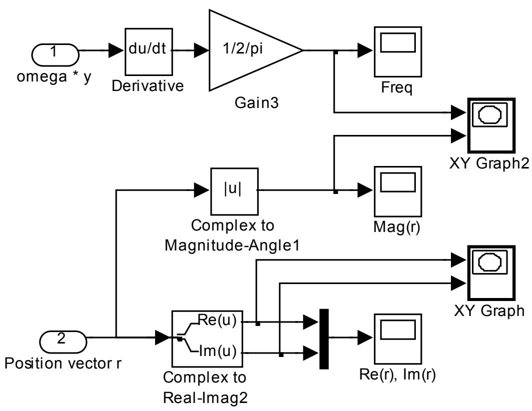

Figure 7: Simulation outputs in the form of time history plots and XY plots

The instantaneous perturbation force frequency is evaluated as the first derivative of the phase, which is a product of the angular velocity and time divided by 2π. Simulations outputs are presented in the form of time history plots and XY plots as it is shown in figure 7.

The parameters K and D, specifying oil film stiffness and damping, are a function of the position vector. The values of these parameters are determined by the oil film thickness. It was proved that the closer position of the journal to the bearing wall and simultaneously the thinner oil film, the greater value of both these parameters. It is assumed that it is possible to approximate both these functions by formulas

$$\begin{aligned} K &= K_0 \Big/ \left(1 - \left(|\mathbf{r}|/e\right)^n\right) \\ D &= D_0 \Big/ \left(1 - \left(|\mathbf{r}|/e\right)^n\right) \end{aligned} \tag{12}$$

where e is a journal bearing clearance. The dependence of the values of the mentioned parameters on the magnitude of the position vector is shown in figure 8.

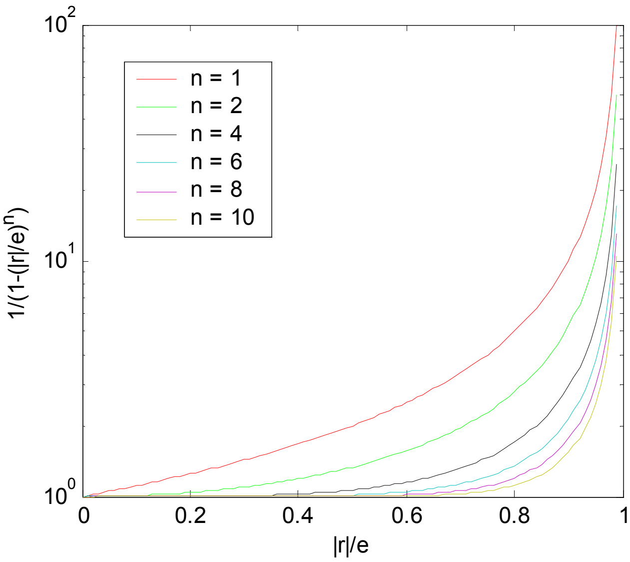

Figure 8: Stiffness and damping parameter value versus journal position

5 Simulation study of the model behavior

The parameters of the tested rotor system are setup to the values as it follows

M = 0.5; % [kg] rotor mass
lam = 0.475; % [-] fluid averaged circumferential velocity ratio (lambda)
K = 20000; % [N/m] oil film stiffness
D = 2000; % [Ns/m] oil film damping coefficient
a = 0.0001; % [m] clearance in the journal bearing

The first simulation test is focused to the resonance behavior of the rotor system with journal bearing. The angular velocity of the rotating force is generated by the Ramp block, whose output is a constantly increasing signal. The steady-state rotation is fixed at the 2400 RPM (40 Hz). The run-up of rotor does not start as a sudden step of RPM to the final value but as a soft start from the stopped rotor position and tracking an exponential function of time with the time constant equaled to 0.5 s. The instantaneous rotational frequency of the perturbation force is evaluated using the first derivative of the rotor rotation angle with respect to time

$$f_{INST} = \frac{1}{2\pi}\frac{d\left(\omega_{RB}\, t\right)}{dt}. \tag{13}$$

As the Ramp block is generating the signal $\omega_{RB} = 2\pi A t$, where A is a constant, the instantaneous rotational frequency is not equal to At, but it is greater twice as At

$$f_{INST} = \frac{1}{2\pi}\omega_{INST} = \frac{1}{2\pi}\frac{d\left(2\pi A t^2\right)}{dt} = 2At \tag{14}$$

The experiment, demonstrating the rotor resonance, is based on using an auxiliary unbalanced disc, which is rotating at the angular frequency ω. The excitation frequency ω is independent on the steady-state rotor angular frequency Ω. The perturbation force is given by unbalance mass m mounted at a radius r_u. To test the rotor resonance using simulation the value of the product mr_u is set to 0.0003 [kgm]. The power of the relative eccentricity in the formulas (12) is chosen 8 to extend the linearity range.

The time history of the real and imaginary parts of the position vector, determining the coordinates of the journal centerline, is shown in figure 9. The magnitude of the journal centerline position vector versus the relative frequency of journal rotation is shown in figure 10. The magnitude reaches the maximum when the frequency of the perturbation force meets approximately half the journal rotational frequency. The behavior of the mathematical model is the same as the true rotor system during experiments.

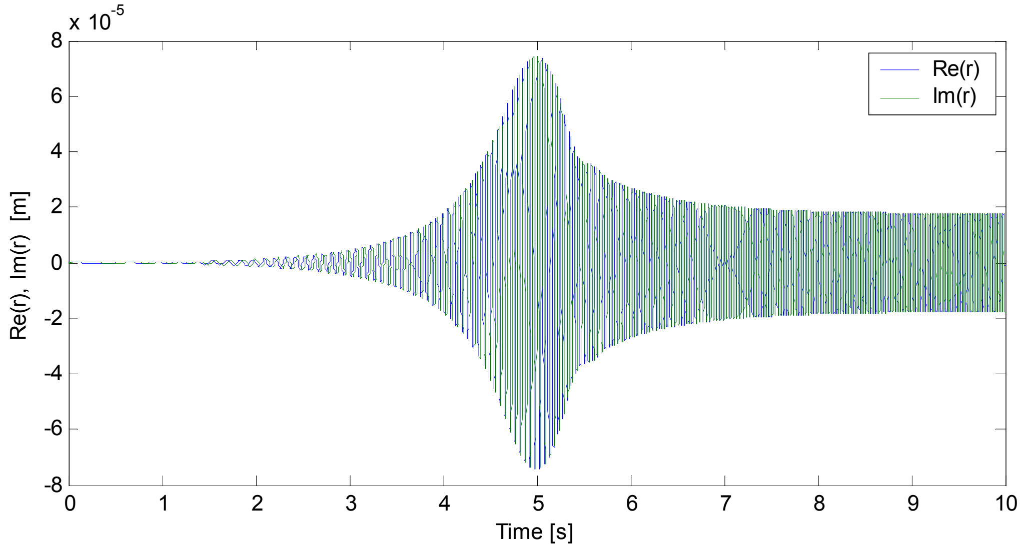

Figure 9: Time history of the journal centerline coordinates

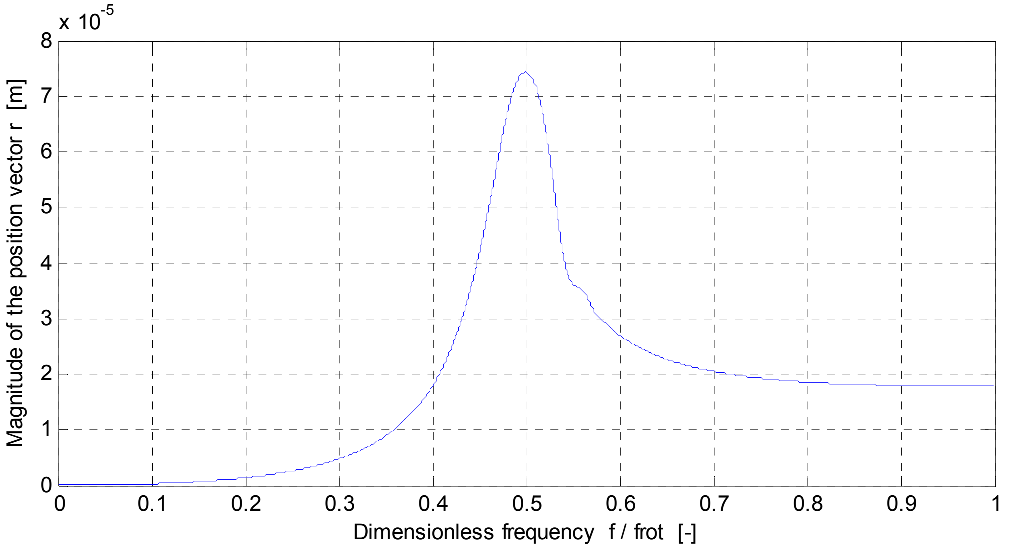

Figure 10: The journal centerline coordinates versus the dimensionless perturbation force rotational frequency related to the rotor rotational frequency

The second simulation test is focused to the rotor instability behavior due to the fluid induced excitation. This phenomenon is known as an oil whirl. As it is stated above, the unstable vibration starts when the rotational speed reaches the certain critical limit value, which is given by the formula

(11). For the parameter values, which are given at the beginning of the chapter, the critical value of rotational frequency f_{Crit} is equal to 67 Hz. For the product mr_u equaled to 0.00005 [kgm] the onset of extensive vibrations is appearing at the rotational frequency 103 Hz as it is shown in figure 11. The simulation tests show that the value of mr_u affects the moment when the instability is started up. The minimal value of the rotational frequency was detected at 95 Hz.

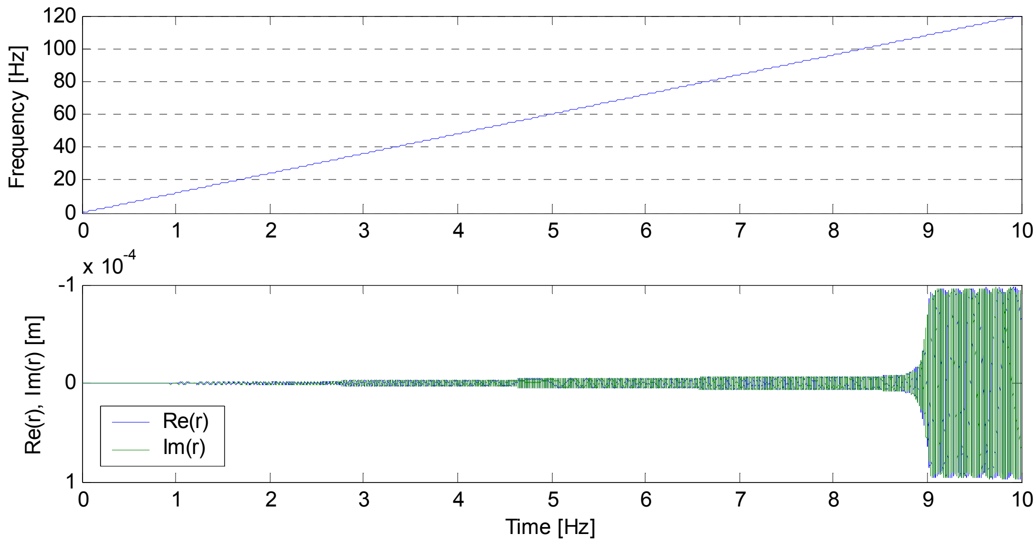

Figure 11: Time history of the rotational frequency and journal centerline coordinates up to the moment when fluid induced vibration starts up

The experiments show that when the rotor is in unstable state (vibration are limited only by the bearing wall), the frequency of vibration is slightly less than half the rotor rotational frequency Ω. The analysis of the stability margin results in the formula (11), which enables to evaluate the frequency of the steady-state vibration as a fraction $\lambda\Omega$ of the rotational frequency Ω. The ZOOMs of the position vector real and imaginary parts just before and after the vibration onset, which are shortened into the time interval of 0.1 s, are shown in figure 12. Comparison of the number of waves in the time intervals of the same length shows that the frequency of vibration drops to half the frequency before the vibration onset. It can be concluded that the behavior of the simulation model and the true rotor system is the same [8, 9].

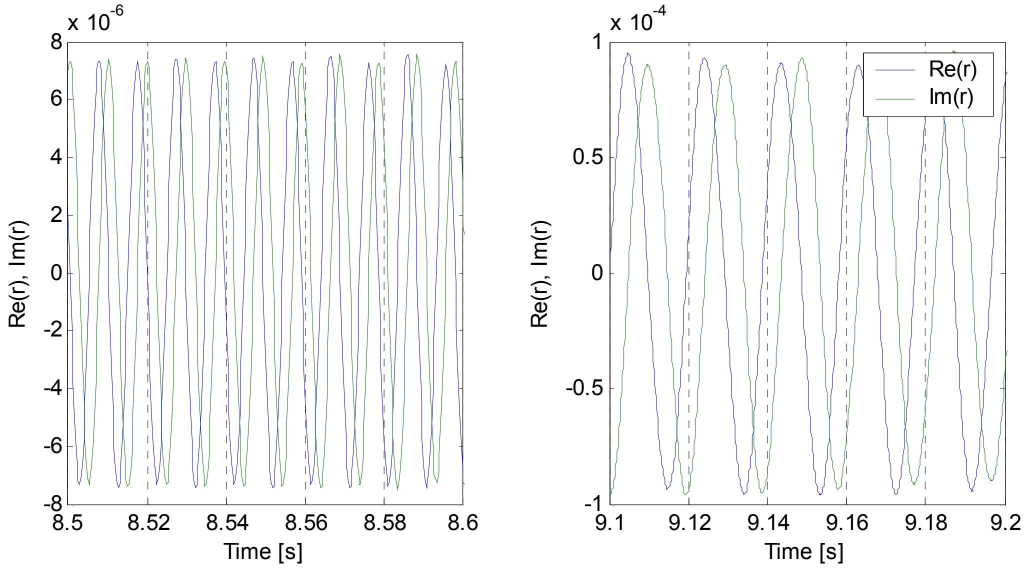

Figure 12: ZOOM of the journal centerline coordinate time history just before and after the vibration onset

All the simulations are performed using the variable integration step and the ODE45 integration method setting.

6 Conclusion

The lumped parameter model of the journal centerline motion in the journal bearing is based on Muszynska's theory. The equation of motion is containing the complex vector and parameters. The main goal of the simulation study was to verify the model principle by comparing simulation results with results of experiments, which are described in many papers, namely the instability of motion and the vibration mode at the non-synchronous perturbation.

The simulation of the rotor system using Simulink results in confirming the agreement between Muszynska's model and experiments.

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