EFFECTIVE AND ROBUST CALCULATION OF MAGNETIC FORCE

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Abstract

Magnetic force between permanent magnets is calculated by the use of simple models of magnetic dipoles or coupled volume and surface currents. The derived integral formulae allow effective programming and calculation by MATLAB. The computation time is acceptable. The uniform magnatisation was supposed and the agreement between theory and experiment is good from technical point of view. Model also calculates correctly effects that are difficult to measure. Therefore, it will be a usefull mean in the design of modern apparatus. The agreement with experiment can be further improved by the model refinement, using the correct space distribution of magnetisation. However, it needs to accelerate the computation, by the use of a cluster, for instance.

1 Introduction

MATLAB is a very simple, versatile and efficient system for numerical computation in electrical engineering. Its main advantages are very effective calculations with matrixes, simple use of complex numbers, intuitive and perfect graphics and a lot of standard and special functions. For special purpose there is a lot of toolboxes. However, usually the basic system is an ideal tool almost for all typical tasks of electrical engineering.

Thanks to the MATLAB advantages we have used this language for the special and non-standard computation in the area of magnetism. Usually, the tasks in this area are solved by the use of Finite Element Method (FEM). However, if the system is relatively simple, the choice of simple model that takes into account all the basic features of the solved problem usually leads to results that are acceptable in praxis.

Such a relatively simple problem is the calculation of repulsive force of permanent magnets of ring shape. The practical motovation is to inovate special textile machine. In that machine main problem is to eliminate high dynamic forces due to the moving parts. Standardly the force is eliminated by the repulsive force generated by resine blocks. The inovation is to replace the resine blocks by permanent magnets and use their repulsive force to perform magnetic breaking.

The simplest way of magnetic breaking realisation is the use two permanent magnets oriented in order to produce repulsive force. Since the equilibrium state of repusive force is instable (labile) in every case, the tangential force, normal to the repulsive one, will be every time present in real devices. The simplest way for mechanical cancellation of the tangential is the use of ring (annulus) permanent magnet shape.

Already from this simple problem descriprion it is clear that the system is not simple and some unwanted effects exist. It is evident that the correct simulation of the system will help in its realization and save expensive experiments. Such simulation of this simplest arrangement using MATLAB will be the subject of the paper. Form the point of view of numerical calculations the paper should demonstrate clearly the possibilities of this language.

Because of its extreme technical importance, a lot of methods for magnetic field and especially magnetic force computation exists, starting from simplest ones based on Ampere's Law, going through physical models that usually use the Biot-Savart Law, and finishing by the application of general modern Finite Element Method. At present time, the Amper's Law application is only the subject of textbooks [1], but both the physical models and FEM are used, for instance physical models are used in Ref. [3], while FEM is a subject of paper [4].

Since the problem is a special one and requires some explanation, relatively extended theory follows. All the formulae used for numerical calculations are presented in this part. Then the model used in MATLAB scripts is outlined. Processing of experimental data follows. In the section dealing with results typical graphs are presented and comparison with experiment is made. The evaluation of results and recommendation for future woork are in the last part of the paper.

2   Theory

In order to calculate the magnetic force, the magnetic field must be calculated first. Then relatively simple formulae can be used for the force calculation. Referring to the application, the only source of magnetic field is a permanent magnet of limited volume V. Elementary (atomic) dipoles are supposed as the source of magentic field. We suppose that current I flows in the (planar) closed loop and the area inside the loop is S. This source of magnetic field is termed magnetic dipole. Its basic parameter, magnetic momentum, is defined by formula

$$\vec{m}_p = I\vec{S} \tag{1}$$

where $\vec{S}$ is the vector of value S, directed normal to the area and its sense is given by flowing current according to clockwise screw motion rule. The elementary magnetic dipole is a limiting case of real dipole for current increasing to infinity and area (and all its dimensions) decreasing to zero in such manner that the product (1) is finite, e.g.

$$\vec{m} = \lim_{\substack{I \to \infty \\ S \to 0}} I\vec{S} \tag{2}$$

Magnetic field of permanent magnet is made by elementary atomic magnetic dipoles. The superposition of elementary dipoles creates the magnetisation $\vec{M}$, which is the basic material parameter. Magnetisation $\vec{M}$ is defined as a magnetic momentum of unit volume of uniform material. Therefore, the elementary volume $\mathrm{d}V$ exhibits an elementary magnetic momentum

$$\mathrm{d}\vec{m} = \vec{M}\mathrm{d}V \tag{3}$$

In all textbooks, for instance [1], it is stressed that there are two approaches for electric or magnetic field decription according to the effects that the field produces:

1. Energy effects — the field potential can be used for potential energy calculation

2. Force actions — the field strength appears in simple formulae for force action

The magnetic field can be described either by the vector potential $\vec{A}$ or by the flux density $\vec{B}$ that appears in simple formula for force acting on electric current in magnetic field. The vector potetial can be used in formulas for energy calculation, but the way is not simple and straightforward in magnetism. The relation between two descriptions is given by formula

$$\vec{B} = \mathrm{rot}\,\vec{A} \tag{4}$$

In principle, there are two basic pocedures for the computation of magnetic field: the diffrenetial and integral ones. The differential approach was described in detail elsewhere [2], therefore the integral approach will be outlined here and used for the calculation of magnetic field. Again, for integral approach, there are two basic possibilities

1. Elementary dipoles

2. Coupled surface and volume currents

Both the methods use two types of coordinates or position vectors: material and field. Material position vector has zero index, $\vec{r}_o = (x_o, y_o, z_o)$, for instace, and it denotes the position of elementary dipole, current element etc. Field vector without any index, $\vec{r} = (x, y, x)$ for instance, defines the point, at which the field is calcutaled. This abbreviation is often used for difference of these vectors

$$\vec{\Delta r} = \vec{r} - \vec{r}_o = (x - x_o, y - y_o, z - z_o) \tag{5}$$

The distance between source (material quantity position) and respose (field quantity position) is denoted by

$$\Delta r = |\vec{r} - \vec{r}_o| = \sqrt{(x - x_o)^2 + (y - y_o)^2 + (z - z_o)^2} \tag{6}$$

In the following parts the model of elementary dipoles and coupled surface and volume currents will be described separately and somewhat in detail.

2.1 Elementary Dipoles

Elementary magnetic dipole is a basic and probably the simplest source of magnetic field. This formula is derived for vector potential in all the textbooks, for instance [1].

$$\vec{A}(\vec{r}) = \frac{\mu_o}{4\pi} \frac{\vec{m}_o \times (\vec{r} - \vec{r}_o)}{|\vec{r} - \vec{r}_o|^3} = \frac{\mu_o}{4\pi} \frac{\vec{m}_o \times \vec{\Delta r}}{(\Delta r)^3} \tag{7}$$

According to our convention, the dipole of momentum $\vec{m}_o$ is in a position given by vector $\vec{r}_o$, while the potential $\vec{A}$ is calculated in the position $\vec{r}$. Other used symbols are defined in (5) and (6). By the application of definition (4) to formula (7) the following formula can be derived for flux density $\vec{B}$ of the elementary dipole of magnetic momentum $\vec{m}_o$ in a position $\vec{r}_o$

$$\vec{B}(\vec{r}) = \frac{\mu_o}{4\pi} \left(\frac{3(\vec{m}_o.(\vec{r} - \vec{r}_o))}{|\vec{r} - \vec{r}_o|^5}(\vec{r} - \vec{r}_o)) - \frac{\vec{m}_o}{|\vec{r} - \vec{r}_o|^3} \right) = \frac{\mu_o}{4\pi} \left(\frac{3\vec{m}_o.\vec{\Delta r}}{(\Delta r)^5} \vec{\Delta r} - \frac{\vec{m}_o}{(\Delta r)^3} \right) \tag{8}$$

Since the magnetic field of permanent magnet is formed by its atomic elementary dipoles, it can be calculeted by the superposition method. The superposition method supposes that in the elementary volume dV_o, given by position vector $\vec{r}_o$, there is a dipole with an elementary moment (see also the equation (3))

$$d\vec{m}_o = \vec{M}(\vec{r}_o) \, dV_o \tag{9}$$

Each elementary dipole of momentum (9) forms the elementary field given by equations (7) and (8). The superposition of elementary fields is realized by integration of the above equations. Vector potential of permament magnet of magnetisation $\vec{M}_o$ in volume V_o can be calculated from formula

$$\vec{A}(\vec{r}) = \frac{\mu_o}{4\pi} \int_{(V_o)} \frac{\vec{M}_o(\vec{r}_o) \times (\vec{r} - \vec{r}_o)}{|\vec{r} - \vec{r}_o|^3} dV_o = \frac{\mu_o}{4\pi} \int_{(V_o)} \frac{\vec{M}_o \times \vec{\Delta r}}{(\Delta r)^3} dV_o \tag{10}$$

The integration is made for the volume V_o, where the magnetic dipoles are present and magnetisation $\vec{M}_o$ is therefore nonzero.

Be the same way we will get the formula for magnetic field flux desity

$$\vec{B}(\vec{r}) = \frac{\mu_o}{4\pi} \int_{(V_o)} \left(\frac{3(\vec{M}_o(\vec{r}_o).(\vec{r} - \vec{r}_o))}{|\vec{r} - \vec{r}_o|^5}(\vec{r} - \vec{r}_o)) - \frac{\vec{M}_o(\vec{r}_o)}{|\vec{r} - \vec{r}_o|^3} \right) dV_o$$
$$= \frac{\mu_o}{4\pi} \int_{(V_o)} \left(\frac{3\vec{M}_o.\vec{\Delta r}}{(\Delta r)^5} \vec{\Delta r} - \frac{\vec{M}_o}{(\Delta r)^3} \right) dV_o \tag{11}$$

From the theroretical point of view, there is no difference between formula (10) and (11), since the second formula was derived from the first one. However, from the computational point of view, the more complicated formula (11) should be used, since it calculates the field directly, while the use of a simple formula (10) requires numerical derivation, which is a source of big numerical errors. This is the reason, why we declare both the formulae.

In the uniform magnetic field of flux density $\vec{B}$ only the mechanical momentum $\vec{M}_f$ given by formula

$$\vec{M}_f = \vec{m} \times \vec{B} \tag{12}$$

acts on the elementary dipole of magnetic momentum $\vec{m}$. If the field is nonuniform, also the force $\vec{F}$ given by formula

$$\vec{F} = (\vec{m}\,\mathrm{grad})\vec{B} \tag{13}$$

acts on elementary magnetic dipole of magnetic momentum $\vec{m}$. The components of the force $\vec{F}$ are calculated from the formula (13) by the equations

$$
\begin{aligned}
F_x &= m_x \frac{\partial B_x}{\partial x} + m_y \frac{\partial B_x}{\partial y} + m_z \frac{\partial B_x}{\partial z} \\
F_y &= m_x \frac{\partial B_y}{\partial x} + m_y \frac{\partial B_y}{\partial y} + m_z \frac{\partial B_y}{\partial z} \\
F_z &= m_x \frac{\partial B_z}{\partial x} + m_y \frac{\partial B_z}{\partial y} + m_z \frac{\partial B_z}{\partial z}
\end{aligned}
\tag{14}
$$

We use the formulae (13) or (14) for the calulation of the force $\vec{F}_e$ acting on elementary dipole of momentum $\vec{m}$ with position vector $\vec{r}$ from the field of another elementary dipole of momentum $\vec{m}_o$ positioned in point of vector $\vec{r}_o$. After the substitution of (8) into (13) or (14) and rearrangements we get the formula

$$
\begin{aligned}
\vec{F}_e &= \frac{\mu_o}{4\pi}\left(\frac{\vec{m}.\vec{m}_o + m_c m_{oc}}{|\vec{r} - \vec{r}_o|^5}(\vec{r} - \vec{r}_o) - \frac{5[\vec{m}_o.(\vec{r} - \vec{r}_o)][\vec{m}.(\vec{r} - \vec{r}_o)]}{|\vec{r} - \vec{r}_o|^7}(\vec{r} - \vec{r}_o) + \frac{3[\vec{m}.(\vec{r} - \vec{r}_o)]}{|\vec{r} - \vec{r}_o|^5}\vec{m}_o \right) \\
&= \frac{\mu_o}{4\pi}\left(\frac{\vec{m}.\vec{m}_o + m_c m_{oc}}{(\Delta r)^5}\vec{\Delta r} - \frac{5[\vec{m}\vec{\Delta r}]^2}{(\Delta r)^7}\vec{\Delta r} + \frac{3[\vec{m}\vec{\Delta r}]}{(\Delta r)^5}\vec{m}_o \right)
\end{aligned}
\tag{15}
$$

where

$$m_c = m_x, \quad m_c = m_y \quad m_c = m_z \quad \text{for} \quad F_{ex}, \quad F_{ey}, \quad F_{ez} \quad \text{respectively} \tag{16}$$

and

$$m_{co} = m_{xo}, \quad m_{co} = m_{yo} \quad m_{co} = m_{zo} \quad \text{for} \quad F_{ex}, \quad F_{ey}, \quad F_{ez} \quad \text{respectively} \tag{17}$$

The force $= \mathrm{d}\vec{F}$ with which the permanent magnet of magnetisation $\vec{M}_o$ in volume V_o acts on elementary dipole of momentum $\vec{m}$ with position vector $\vec{r}$ can be found analogically by the use the formulae (11) instead of (8) After the substitution of (11) into (13) or (14) and rearrangements we get the formula

$$
\begin{aligned}
\mathrm{d}\vec{F} &= \frac{\mu_o}{4\pi}\int_{(V_o)} \frac{\vec{m}.\vec{M}_o(\vec{r}_o) + m_c m_{oc}}{|\vec{r} - \vec{r}_o|^5}(\vec{r} - \vec{r}_o)\mathrm{d}V_o \\
&\quad - \frac{\mu_o}{4\pi}\int_{(V_o)} \frac{5[\vec{m}.(\vec{r} - \vec{r}_o)][\vec{M}_o(\vec{r}_o).(\vec{r} - \vec{r}_o)]}{|\vec{r} - \vec{r}_o|^7}(\vec{r} - \vec{r}_o)\mathrm{d}V_o \\
&\quad + \frac{\mu_o}{4\pi}\int_{(V_o)} \frac{3[\vec{m}.(\vec{r} - \vec{r}_o)]}{|\vec{r} - \vec{r}_o|^5}\vec{M}_o(\vec{r}_o)\mathrm{d}V_o \\
&= \frac{\mu_o}{4\pi}\int_{(V_o)}\left(\frac{\vec{m}.\vec{M}_o + m_c m_{oc}}{(\Delta r)^5}\vec{\Delta r} - \frac{5[\vec{m}.\vec{\Delta r}][\vec{M}_o.\vec{\Delta r}]}{(\Delta r)^7}\vec{\Delta r} + \frac{3[\vec{m}.\vec{\Delta r}]}{(\Delta r)^5}\vec{M}_o \right)\mathrm{d}Vo
\end{aligned}
\tag{18}
$$

The meaning of symbols m_c and m_{co} is in equations (16) and (17).

The total force, by which two permanent magnets act one another, can be got from the formula (18). Suppose that the permanent magnet of magnetisation $\vec{M}_o$ and volume V_o acts on another permanent magnet of magnetisation $\vec{M}$ and volume V. The magnetic force can be got by integration of elementary forces in (18)

$$
\begin{aligned}
\vec{F} = \int_{(V)} \mathrm{d}\vec{F}\mathrm{d}V &= \frac{\mu_o}{4\pi} \int_{(V)} \left(\int_{(V_o)} \frac{\vec{m}.\vec{M}_o(\vec{r}_o) + m_c m_{oc}}{|\vec{r} - \vec{r}_o|^5} (\vec{r} - \vec{r}_o)\, \mathrm{d}V_o \right) \mathrm{d}V \\
&- \frac{\mu_o}{4\pi} \int_{(V)} \left(\int_{(V_o)} \frac{5[\vec{m}.(\vec{r} - \vec{r}_o)][\vec{M}_o(\vec{r}_o).(\vec{r} - \vec{r}_o)]}{|\vec{r} - \vec{r}_o|^7} (\vec{r} - \vec{r}_o)\, \mathrm{d}V_o \right) \mathrm{d}V \\
&+ \frac{\mu_o}{4\pi} \int_{(V)} \left(\int_{(V_o)} \frac{3[\vec{m}.(\vec{r} - \vec{r}_o)]}{|\vec{r} - \vec{r}_o|^5} \vec{M}_o(\vec{r}_o)\, \mathrm{d}V_o \right) \mathrm{d}V \\
&= \frac{\mu_o}{4\pi} \int_{(V)} \left(\int_{(V_o)} \frac{\vec{m}.\vec{M}_o + m_c m_{oc}}{(\Delta r)^5} \vec{\Delta r}\, \mathrm{d}Vo \right) \mathrm{d}V \\
&- \frac{\mu_o}{4\pi} \int_{(V)} \left(\int_{(V_o)} \frac{5[\vec{m}.\vec{\Delta r}][\vec{M}_o.\vec{\Delta r}]}{(\Delta r)^7} \vec{\Delta r}\, \mathrm{d}Vo \right) \mathrm{d}V \\
&+ \frac{\mu_o}{4\pi} \int_{(V)} \left(\int_{(V_o)} \frac{3[\vec{m}.\vec{\Delta r}]}{(\Delta r)^5} \vec{M}_o\, \mathrm{d}Vo \right) \mathrm{d}V
\end{aligned}
\tag{19}
$$

This symbolic equation can be used for numerical calculation of magnetic force between two permanent magnets.

2.2 Coupled surface and volume currents

This method is a quite different one. The textbooks on electromagnetics, for instance [1], show that the effect of elementary magnetic dipoles is equivalent to the coupled currents that flow both on the surface and in the volume of magnitized media, in general. For the density of coupled volume currents the following equation is valid

$$
\vec{i}_m(\vec{r}_o) = \mathrm{rot}\,\vec{M}(\vec{r}_o)
\tag{20}
$$

The derivations in the operation rot are made by the material coordinates, which is symbolically denoted by material position vector $\vec{r}_o$. The coupled surface currents have the density

$$
\vec{j}_m(\vec{r}_o) = \mathrm{Rot}\vec{M}(\vec{r}_o) = \vec{n} \times (\vec{M}_2(\vec{r}_o) - \vec{M}_1(\vec{r}_o)) = -\vec{n} \times M(\vec{r}_o)
\tag{21}
$$

where $\vec{n}$ is the surface normal from the magnetic body (index 1) to vacuum (index 2). In vacuum the magentisation is zero, $\vec{M}_2(\vec{r}_o) = 0$[1], in the body $\vec{M}_1(\vec{r}_o) = \vec{M}(\vec{r}_o)$.

As soon as both the surafce and volume current densities are given, the magnetic field flux density can be calcuted by the Biot-Savart Law. Coupled volume currents produce the field

$$
\vec{B}(\vec{r}) = \frac{\mu_o}{4\pi} \int_{(V)} \frac{\vec{i}_m(\vec{r}_o) \times (\vec{r} - \vec{r}_o)}{|\vec{r} - \vec{r}_o|^3}\, \mathrm{d}V = \frac{\mu_o}{4\pi} \int_{(V)} \frac{\vec{i}_m \times \vec{\Delta r}}{(\Delta r)^3}\, \mathrm{d}V
\tag{22}
$$

while the field from coupled surface currents is given by formula

$$
\vec{B}(\vec{r}) = \frac{\mu_o}{4\pi} \int_{(S)} \frac{\vec{j}_m(\vec{r}_o) \times (\vec{r} - \vec{r}_o)}{|\vec{r} - \vec{r}_o|^3}\, \mathrm{d}S = \frac{\mu_o}{4\pi} \int_{(S)} \frac{\vec{j}_m \times \vec{\Delta r}}{(\Delta r)^3}\, \mathrm{d}S
\tag{23}
$$

---

[1]Strictly speaking, the material vector $\vec{r}_o$ should not be used outside the body.

where S is the surface of magnetised body.

If the total volume or surface coupled current can be approximated by the curve current I, the simplest formula can be used

$$\vec{B}(\vec{r}) = \frac{\mu_o}{4\pi} I \int_{(c)} \frac{\vec{t}_o \times (\vec{r} - \vec{r}_o)}{|\vec{r} - \vec{r}_o|^3} \, \mathrm{d}l = \frac{\mu_o}{4\pi} I \int_{(c)} \frac{\vec{t}_o \times \vec{\Delta r}}{(\Delta r)^3} \, \mathrm{d}l \tag{24}$$

where $\vec{t}_o$ is the tangential unit vector to the element $\mathrm{d}l$ in the direction of current on the curve c in which the current I flows. This approximation can be used in practical calculation.

Very important conlusion follows from the use of the method of coupled currents. The magnetic medium can be exactly replaced by coupled current (volume and surface) flowing in vaccuum.

The magnetic force can be calCulated using the modifield fomula for Lorentz force. The force aCting on the element of volume current $\vec{i}_m \mathrm{d}V$, element of surface current $\vec{j}_m \mathrm{d}S$ and element of curve current $I\vec{\mathrm{d}l}$ are given by respectively formulae

$$\mathrm{d}\vec{F}_v = (\vec{i}_m \times \vec{B})\mathrm{d}V$$
$$\mathrm{d}\vec{F}_s = (\vec{j}_m \times \vec{B})\mathrm{d}S \tag{25}$$
$$\mathrm{d}\vec{F}_c = I(\vec{\mathrm{d}l} \times \vec{B})$$

After the substitution from coresponding equations (22) and (23) into (25) we will get integral formulae for elementary force due to the permanent magnet of volume V_o anD surface S_o, which is modelled by volume coupled currents $\vec{i}_{om}$ and surface coupled currents $\vec{j}_{om}$

$$\mathrm{d}\vec{F} = \frac{\mu_o}{4\pi} \vec{i}_m(\vec{r}) \times \left(\int_{(V_o)} \frac{\vec{i}_{om}(\vec{r}_o) \times (\vec{r} - \vec{r}_o)}{|\vec{r} - \vec{r}_o|^3} \, \mathrm{d}V_o \right) \mathrm{d}V$$
$$+ \frac{\mu_o}{4\pi} \vec{j}_m(\vec{r}) \times \left(\int_{(S_o)} \frac{\vec{j}_{om}(\vec{r}_o) \times (\vec{r} - \vec{r}_o)}{|\vec{r} - \vec{r}_o|^3} \mathrm{d}S_o) \right) \mathrm{d}S \tag{26}$$

The total force, by which the permanent magnet of volume V_o and surface S_o, modelled by volume coupled currents of density $\vec{i}_{om}$ and surface coupled currents of density $\vec{j}_{om}$, acts on the permanent magnet of volume V and surface S, modelled by volume coupled currents of density $\vec{i}_m$ and surface coupled currents of density $\vec{j}_m$, is obtained by the integration of formula (26) on the forced magnet

$$\vec{F} = \frac{\mu_o}{4\pi} (\int_{(V)} \vec{i}_m(\vec{r}) \times \left(\int_{(V_o)} \frac{\vec{i}_{om}(\vec{r}_o) \times (\vec{r} - \vec{r}_o)}{|\vec{r} - \vec{r}_o|^3} \, \mathrm{d}V_o \right) \mathrm{d}V$$
$$+ \frac{\mu_o}{4\pi} \int_{(S)} \vec{j}_m(\vec{r}) \times \left(\int_{(S_o)} \frac{\vec{j}_{om}(\vec{r}_o) \times (\vec{r} - \vec{r}_o)}{|\vec{r} - \vec{r}_o|^3} \mathrm{d}S_o) \right) \mathrm{d}S \tag{27}$$

For the approximate calculation the substitution from equation (24) into the last formula of (25) leads to integral formula for elementary force due to the approximate current I_o flowing on curve c_o applied to the current element with current I and direction $\vec{\mathrm{d}l}$

$$\mathrm{d}\vec{F} = \frac{\mu_o}{4\pi} I I_o \vec{\mathrm{d}l} \times \int_{(c_o)} \frac{(\vec{\mathrm{d}l}_o \times (\vec{r} - \vec{r}_o)}{|\vec{r} - \vec{r}_o|^3} \tag{28}$$

The total force between two approximated currents, I on loop c and I_o on loop c_o can be got by integration of (28)

$$\vec{F} = \frac{\mu_o}{4\pi} I I_o \int_{(c)} \vec{\mathrm{d}l} \times \left(\int_{(c_o)} \frac{(\vec{\mathrm{d}l}_o \times (\vec{r} - \vec{r}_o)}{|\vec{r} - \vec{r}_o|^3} \right) \tag{29}$$

Especially formula (27) wil be used for calculation of the force between two permanent magnets. On the other hand, the formula (29) can be used for calculation of the force action between two loops, irrespective of the nature of currents.

3   Model

The used ring magnet was polarized in the direction of its thickness. In this practical case we have made the basic assumption that the magnetization is uniform. Unfortunately, the supplier data sheet also contains only one value of magnetization, more correctly its low and high limits. This assumption is not valid exactly, since the magnetization is strictly uniform only in an ellipsoid. Therefore, the uniform magnetization can be supposed with small errrors, if the magnet cross-section is circle or ellipse. In the case of rectangular cross-section, the magnetization will not be uniform at least near the edges.

At present time we have limited to the use of the model of coupled currents, since it apperas to be simpler for numerial calculations. Moreover, the use of (free) currents is typical in magnetic problems and calculations. Theerfore, the coupled current model better corresponds to praxis and it is also better understandable.
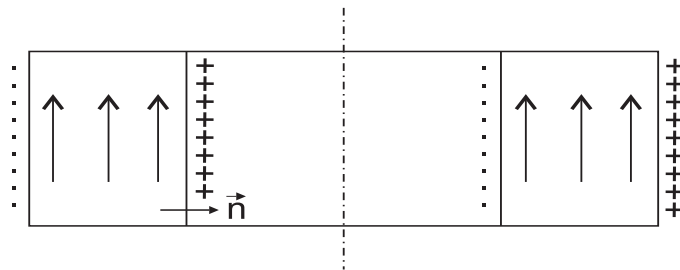
Figure 1: Model of coupled surface currents

For supposed uniform magnetization no volume coupled currents are present and the the surface current density is uniform. The surface currents flow on the circular surfaces of ring magnets in opposite directions as it is schematically shown by dots and crosses in Fig. 1. Since the magnetization is supposed to be uniform inside all the body volume, total 2 surface currents of opposite direction of constant density exist and no volume currents are present. The magnetic field can be calculated according to formula (23). If the surface currents are approximated by one curve current near the central part of the magnet surface, the simplest formula for calculation (24) can be used. The total force can be calculated by the second member of formula (27), as only coupled surface currents exist. If the surface coupled currents are approxiamted by curve currents, simple formula (29) is used.

Becuase of cylinder geometry, the cylindrial coordinates, radius r and azimuth angle α in the magnet plane and z-coordinate along the magnet axis are used. The integration is performed numerically by summation. Typical values of resolution are about one degree for azimuth and about 10 values for z-coordinate. Two values of radius (inner and outer) are used. One point of magnetic field requires two nested cycles calcualiton that are calculated twice for two values of the radius. If the force is calculated, the numeber on nested cycles inceraseas to four ones, if we neglect the cycles for radius. Therefore, the required computation time for the force can be considerably high. Some strategy is used for speeding the calculation, for esxample field and elementary force symmetry, but the details are omitted here.

4   Experiment

Each model has practical meaning only in the case, if it is verified by experiment. In the ring magnet appliaction two types of experiment were made.

1. Magnetic field measurement

2. Repuslive force measurement

Samples were permanent magnets with magnetisation of 1.2 T approximately. The magnet shape was a ring, inner diameter of 25 mm, outer diameter of 70 mm and height of 4 mm. The magnetic field was merasured on one magnet. For force measurements combinations of 1, 2 and 3 magnets accordingly oriented were used. Optimum results were obtained for 3 magnets, they will be presented here.

Both the experiments were made on fully automated apparatus. The measurement of magnetic filed consisted of a small number of points, as the Hall probe dimensions are relatively large. Therefore, instead of the strict local value, the average value of magnetic flux density was measured. Because of their small number, no direct data processing was necessary.

The repulsive force was measured both in the forward and backward motion of magnets. More than 2000 data points was automatically measured and written into computer file. Since some spred of points appeared, the immediate data processing was necessary. It consists of three procedures: sorting, averaging and curve fitting. The sorting ensured that the data course is monotonic, if eperimental errors are neglected. Then the data was grouped into about 100 groups and the average value and standard deviation were calculated for each group. The last step was the force approxiamtion (F_a)by negative powers of distance z, e.g.

$$F_a(z) = \sum_{i=0}^{i=N} A_i z^{-i} = \sum_{i=0}^{i=N} \frac{A_i}{z^i} \tag{30}$$

Several following figures illustrate the force data processing. Approximation of raw data is in Fig. 2. Average values of 100 groups of data points and their approximation by polynom of negative powers of maximum degree of 8 are in Fig. 3. Approximating coefficients are in Fig. 4. The dominant powers are -3, -4 and -5. Relative errors for individual groups are in Fig. 5. For smal distances the relative error is acceptable, less than 10 %, however, near the end of measured interval, the error is very high, much more than 50 %.

The approximation by negative powers was made by the standard MATLAB function `polyfit`. Vector `z` of experimental z-coordinates was replaced by their reciprocal values `zrec=1./z`. The graph in Fig. 2 or Fig. 3 is the output of these commands

```
coef= polyfit(F,1./z,8);
polyval(coef,1./z);
plot(z,F);
```

The programming in MATLAB is really very simple.

5   Results

Basic and key results will be presented in a graphical form predominantly. We prefer the comparison of calculation with epxeriment whenever it is possible. The comparison of calculated and measured magnetic flux density for three distances from magnet surface is in Fig. 6 thru Fig. 8. The comparison with experiment was made for three distances from magnet surface: zero distance in Fig. 6, medium distance in Fig. 7 and realtively large distance in Fig. 8. Just on the surface of the magnet the agreement with experiment is not good at edges (see Fig. 60. In other points on the surafce there is a relatively good agreement. For small distance from the magnet (3 mm in Fig. 7) the agreement is good but small difference is still visible. The agreement further improves with increasing distance from magnet, for distance of 5 mm. (Fig. 8) the deviations are practically due to the experimental errors.
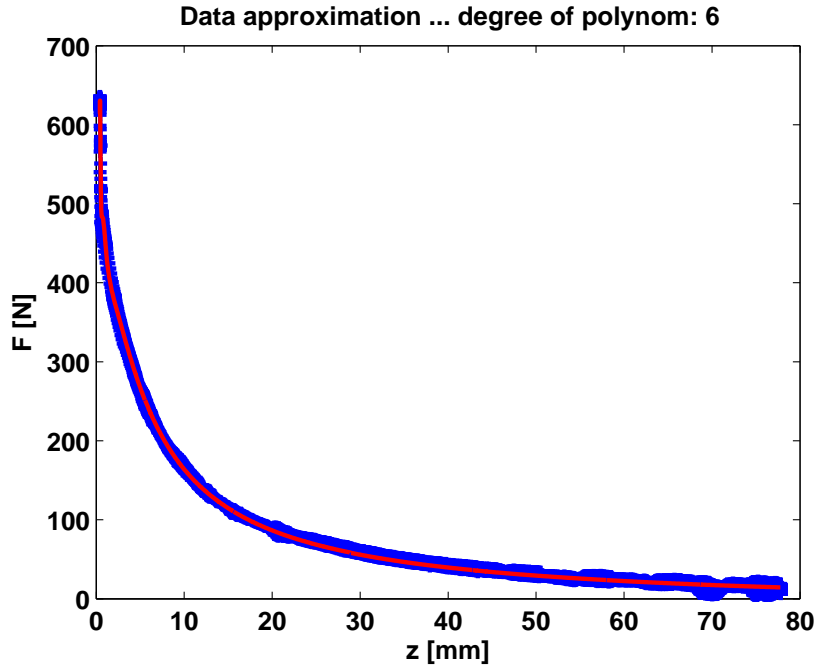
Figure 2: All measured data and their approximation

The comparison of calculeted force and its experimental measurement is in several following figures. In Fig. 9 the calculation results and averaged experiment values are compared. The agreement is relatively good. In the next Fig. 10 the calculated curve and curve approximating averaged experiment values are presented. A systemetic deviation is visible. For lower distances the calculation is above the experiment, for higher distances the calculated force is lower than the measured one. Nevertheess, the agreement between theory and experiment is again relatively good.

The force maximum and minimum values differ more than one order, therefore, the comparison on graphs with linear force scales, as those in Fig. 9 or Fig. 10 has low resolution for small force values. More information can be got from relative deviation that is defined as

$$\delta F = 100\frac{|F_c - F_{exp}|}{F_c} \tag{31}$$

where F_c is the calculated force and F_{exp} is the experimantal value of force, which is either avareged point value or point on approximating curve.

The relative deviation for averaged experimental points is in Fig. 11. For high distance the deviation is very high, it is due to the experimental inaccuracy. For lower values of distace (less than about 50 mm) the deviation is between +20 % and -40 %. The relative deviation between calculetd curve and curve approximating experimental points is in Fig. 12. The deviation varies between +20 % and -60 %. However the magnetisation had to inccrease to 1.3 T (e.g. by 0.1 T) in order to shift the force graphs vertically, by about 20 %. This change can be explained by the spread of magnet parameters. The flux density measuremetrs used another sample. No repetitive measurements on the same samples and the same measurements on different samples were made.

If the magnets are perfectly centered, only axial force axists, which was presented in Fig. 9 or Fig. 10. No mechanical momentum exists. The equibrium is lable, however, any small deviation from exact centering leads to a radial force and mechanical momentum. The radial force for relatively small deviation of 0.3 mm (the outer ring diameter is 70 mm) in the direction of X-axis leads to a relatively large radial force presented by the graph in Fig. 13. This radial force leads to the mechanical momentum along Y-axis. Its values are in Fig. 14. This momentum
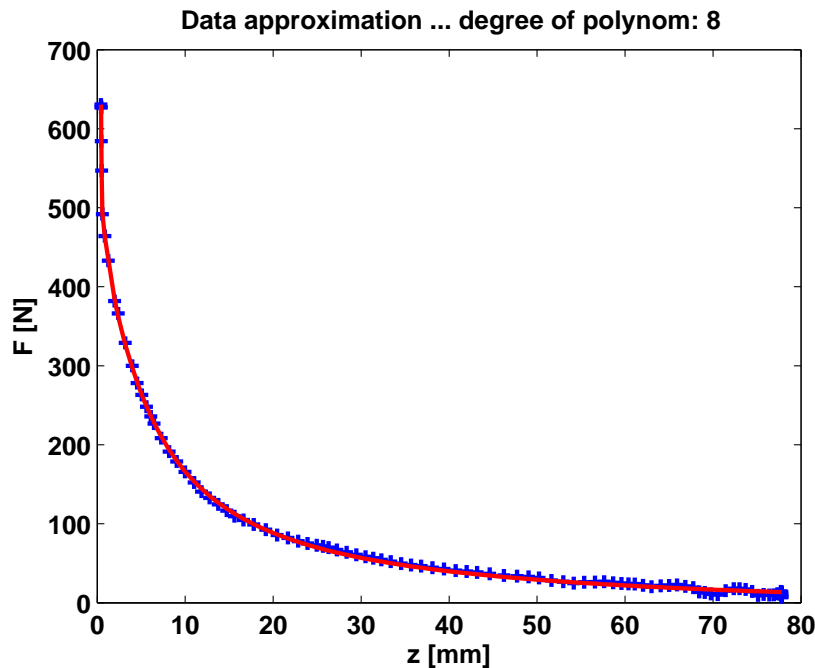
Figure 3: Average values of data and their aproximation

has tendency to rotate the magnets by 180° in order to replace repulsive force by the attractive one. The equilibrium is therefore unstable in any case. No measurements of this force were made, because of their complexity, difficulty and innacuracy.

The total coupled current is shown in the Fig. 13 and Fig. 14. The current is very high, very difficult realizable practically. It confirms the quality of the permanent magnets.

6   Discussion

Insted of finite method application, the simple models were used both for the field of permanent magnet calculation and for computation of repulsive force. Two different models were created. The first model is based on the elementary magnetic dipole interaction. Its base part is the computation of the field produced by the elementary magnetic dipole. The second model replaces the magnet by coupled volume and surface currents. Basic integral equations were derived for the field and force calculation for both the models.

The advantage the of dipole model is that it is better suited for programming. There is a lot of scalar vector products, see formula (19), for instance, that do not requre any cycles in MATLAB. But the numerical integration must be performed over the whole magnet volume, therefore, the force caluclation requres six nested cycles. On the other hand, any space distribution of magnetization is possible, and the computation time does not increease at all.

The coupled current model can ensure much faster computation. However, the programming is little complicated, since there are vector products, see equation (27), for instance. If the magnetisation can be supposed as uniform, the coupled current model requires four nested cycles that shloud be performed twice, for inner and outer ring surface. For non-uniform magnetisation, the four nested cycles can remain, if the non-uniformity can be modelled by surface coupled currents. However, if coupled volume currents are necessary, six nested cycles should be used. Fortunately, coupled volume currents should usually flow only in a limited part of magnet volume.

The numerical integration was performed by MATLAB. The coupled current model was preferred from the only practical reason that the calculation is faster, as it was explained in
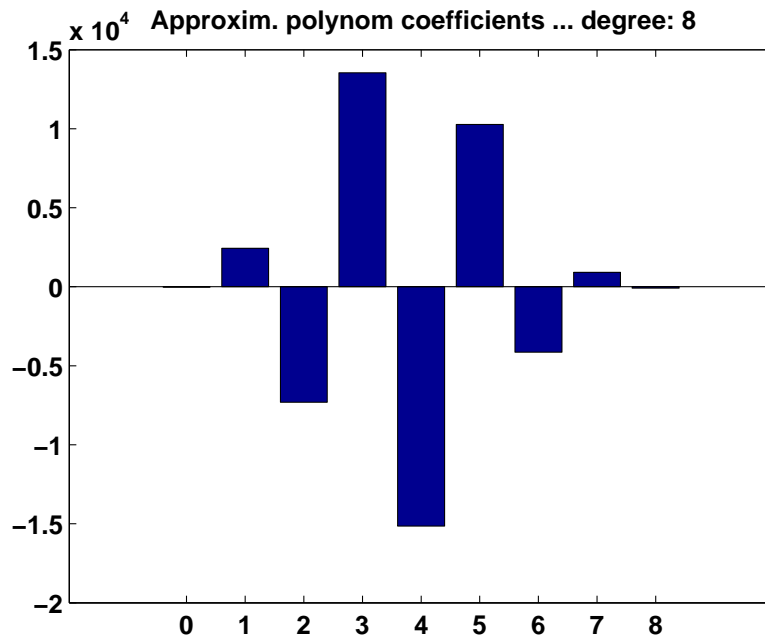
Figure 4: Approximating polynom coefficients in bar graph

previous paragraph. Several scripts of different strategy and complexity were written in order to improve the accuracy and the speed and to verify algorithms. The computation time differs from about half minute to about 15 minutes. It depends mainly on the numebr of elements used in numerical integration, further on the strategy, since the numerical integration can be performed by several ways. Results shown in previous sections were made by the simplest script.

Both the physical models and Finite Element Method (FEM) are used in magnetic computations at present time. For relatively simple structures physical models are used, for instance in Ref. [2], while for complicated systems the FEM is preferred (Example of optimisation based on FEM is in Ref. [3].). Of course, the best choice is the analytical solution. Unfortunately, our system is too complicated, to be solved analytically, either by differential equations or by integral formulae. On the other hand the system is simple enough to allow the modelling by relatively simple models. Two basic moddels (elementary magnets and coupled currents) were derived in detail.

Another general solution is FEM. The only advatage of FEM is that is can be applied to any system in principle and the application is relatively simple, since a lot of commercial systems exist. But there are serious disadvantages. Serious erros in the task definition cannot be protected by the scommertial system. The user has no check at the calcualtion phase and cannot verify it. The results are approximate and the output is in a set of points selected by FEM, their position cannot be affected by the user.

The physical model, on the other hand, is opposite to FEM in many apects. The disadvantage is that the user must create the model and find integral formulae describing it. The user alse must program the numeric integration. But the user has full check on program and calculation. The user can verify the outpus by many ways. The results are exactly in points selected by the user and their accuracy can be improved be increasing the number of elements used in numerical integration. Unfortunately, also the computation time increases rapidly, which is the unwated effect. Therefore, compromise between accuracy and time should be found.

Of course, the best case is analytical soulution. It is accurate at all the points.

As it was presented in section Results, the agreement with experiment is acceptable from the technical point of view. Extensive experiments were not performed, only informative ones
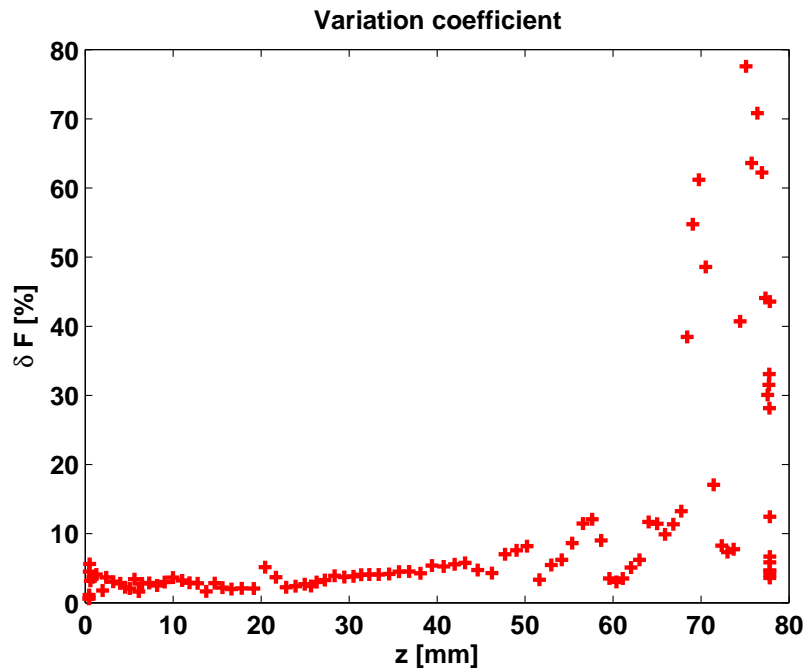
Figure 5: Relative errors for data groups

were realised, in order to verify experimentally new principle and method. No repetitive measurements were performed. Therefore, the data may contain relatively large error, it may be about 10 %. Then the calculation innacuracy is in the range of 20 % in the area of technical interest, which is fully acceptable in the system simulation. Many other important results, as radial force or momentums can be obtained. They can help in the proper system design.

Nevertheless, there is a systematic deviation between experiment and theory both in the magnetic field and repulsive force distribution. Probably, it is a consequence of the simplest used model that suppose uniform magnetization. Further model improvements require finding of relibale magnetisation distribution.

Therefore, irrespective of the used method (physical model or FEM), the key question is the correct magnetization distribution. The magnetization cannot be measured directly; the only information is from the field at the surface and near it. Unfortunately, the measuring accuracy of Hall probe is relatively low and only average values are measured. Abrupt changes as in Fig 6 cannot be measured correctly, due to the Hall probe integration effect, for instance.

In general, the correct magnetization can be found from experimental data by the method of trials and errors. Correction for magnetization, or coupled currents that are equivalent to the magnetization, can be found from the differences between experiment and model. It requires a lot of creative work. In order to make corrections in real time, a very fast numeric integration is necessary. The optimum solution is to use parallel working computers, the cluster. The present task of identical numeric integration in many points makes the cluster programming relatively simple. The task is an ideal one for distributed calculation.

7 Conclusions

We have shown that universal language MATLAB can be very effectively used for a special technical calculations. The advantage is a very simple programming, the only disadvantage is a little low speed of computation, if we need detailed and accurate outputs. But it can be improved by the use of parallel computing.

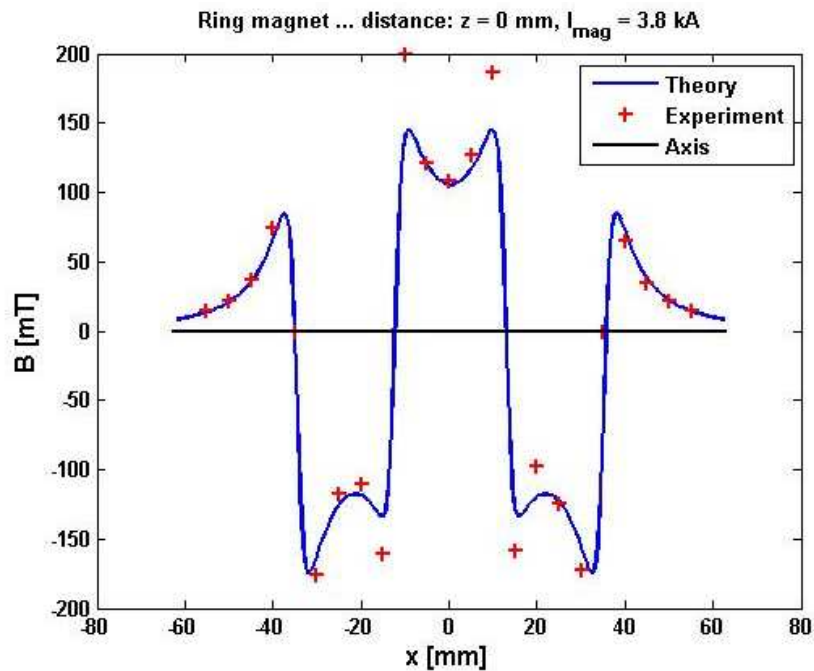In the problem solution two physical models were derived and one of them (model of

Figure 6: Comparison of calculated and measured magnetic field on the magnet surafce

coupled currents) was used for extensive calculations. The model of coupled current is probably the best model for a given arrangement. It main advantage is that the magnetic material is replaced by currents in vacuum.

Since the agreement with experiment is good, the model can predict correctly a lot of important features or real apparatus and save a lot of experimental work. Radial forces, presented in section Results, are an example of such important results that are difficult to measure. Therefore, the present system can be used in computer aided design of apparatus.

The model improvement needs to find a correct space distribution of magnetisation. Since the only solution is by the method of trials and errors, fast compotation is necessary to apply the method effectively. The distributed calculation on a cluster is necessary.

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[3] Q. L. Peng, S. M. McCurry, and J. M. D. Coley. *Axial Magnetic field prouduced by axially and radially magnetized rings.* J. of Magnetism and Magnetic Materials, vol. 268, 2004, pp. 165-169.

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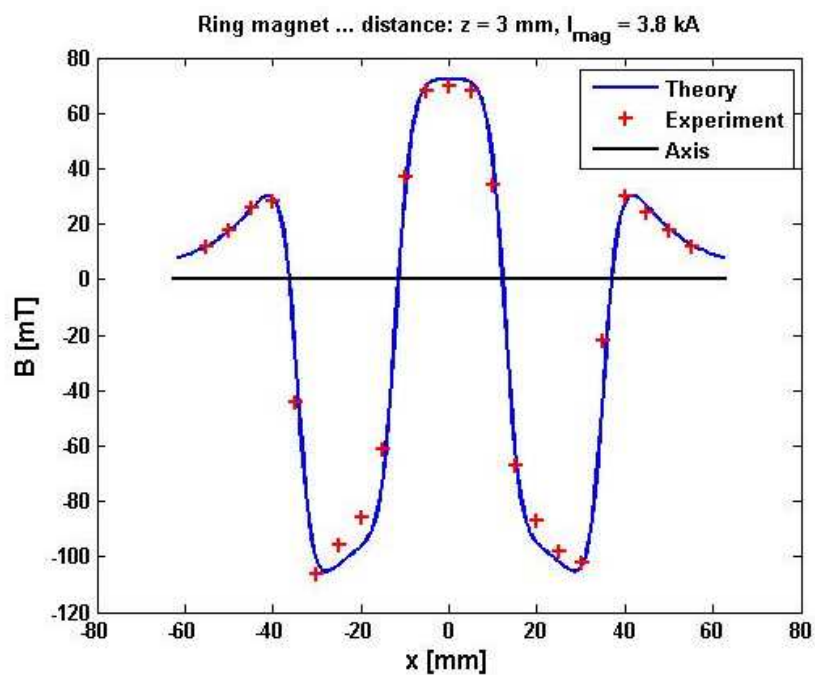
Figure 7: Comparison of calculated and measured magnetic field at distance of 3 mm from magnet

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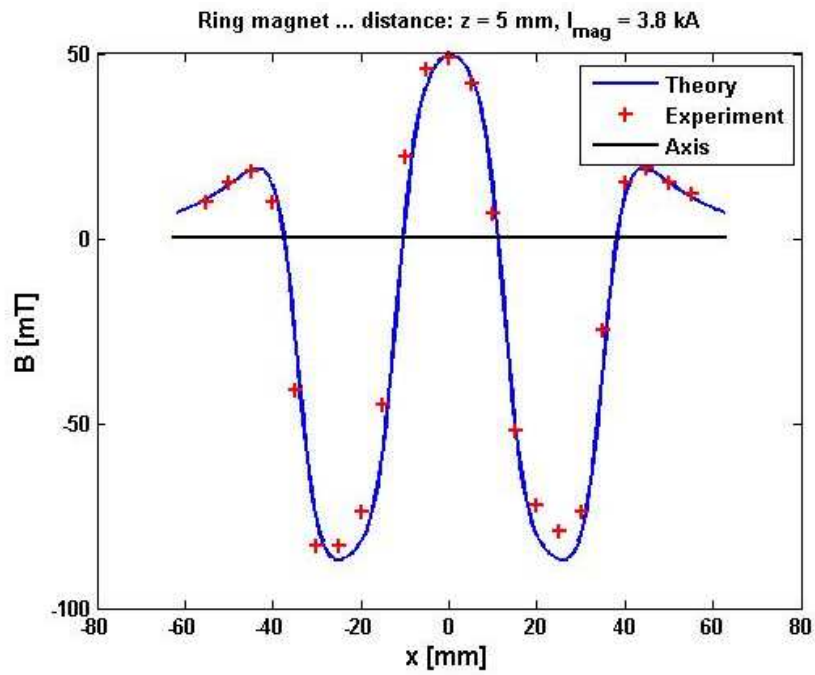
Figure 8: Comparison of calculated and measured magnetic field at distance of 5 mm from magnet
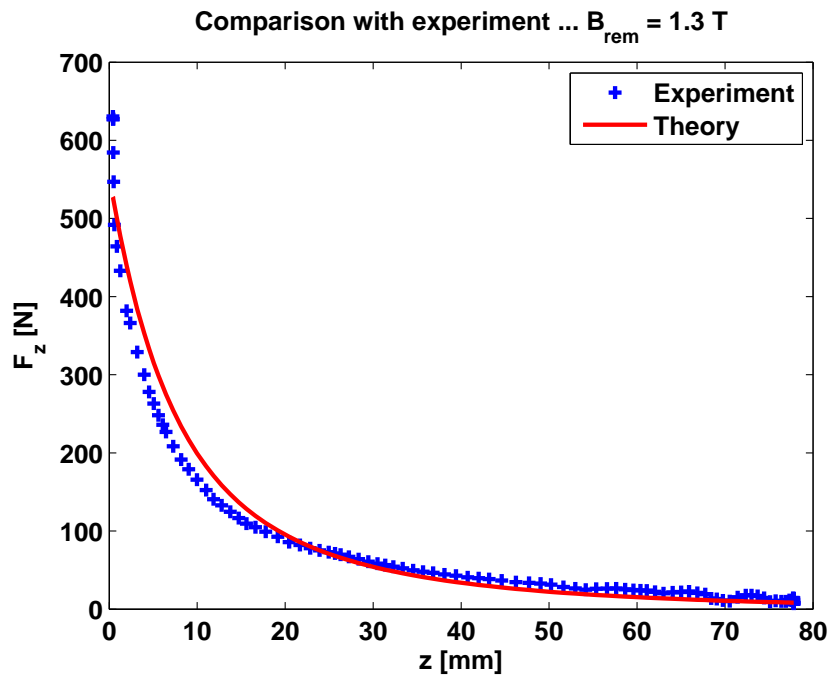
Figure 9: Comparison of calculated force and the averaged experimental data
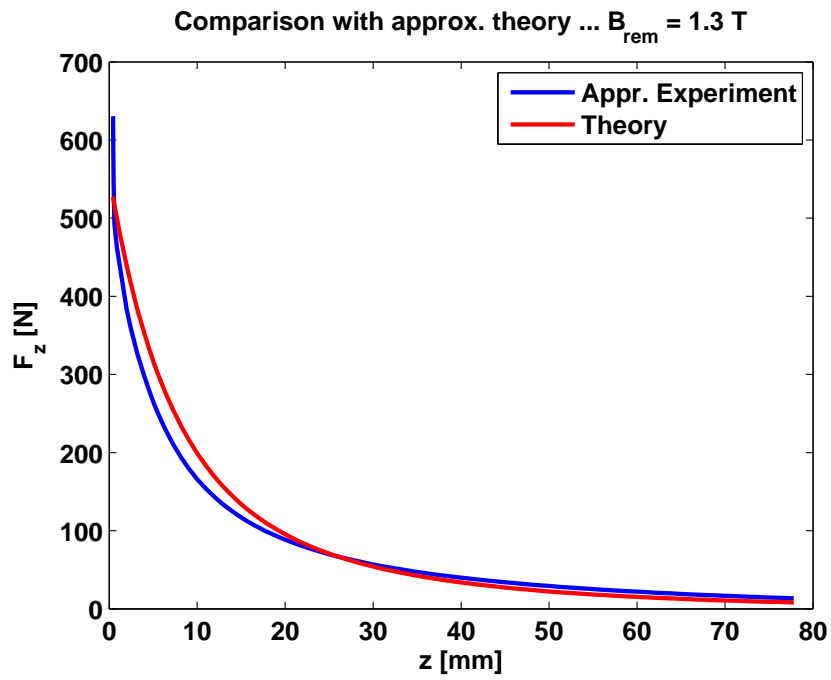
Figure 10: Comparison of calculated force with curve aprroximating experimental data
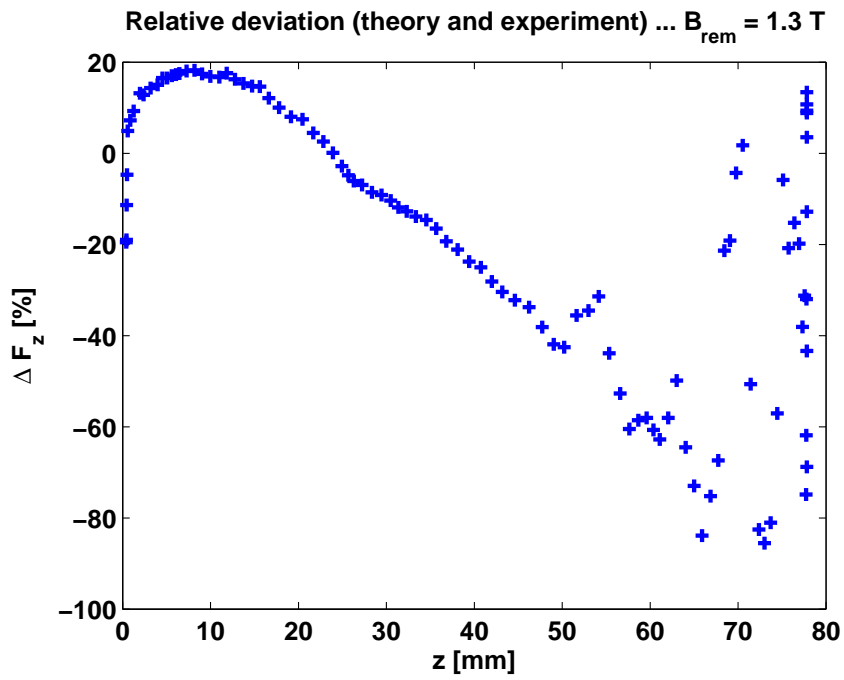

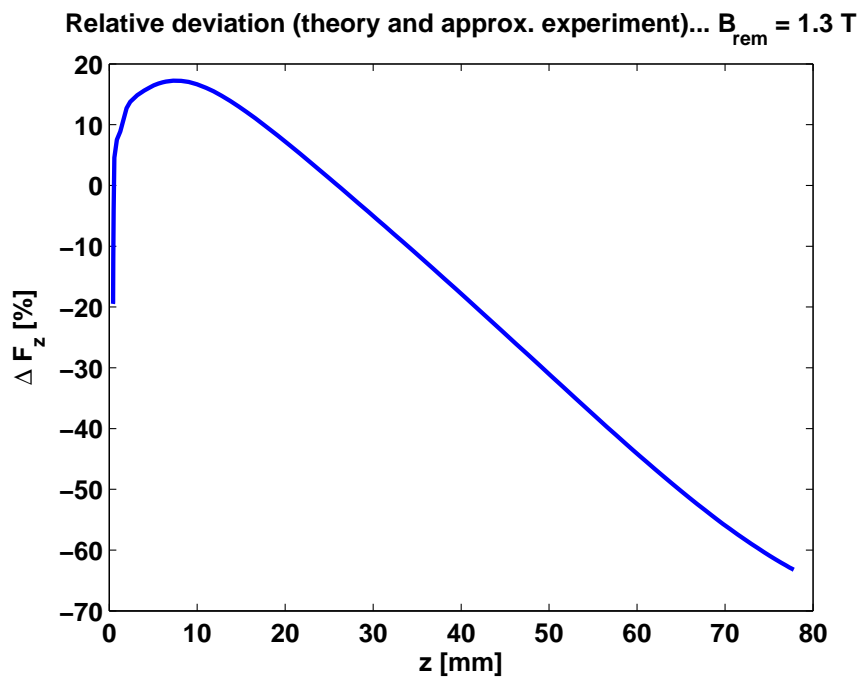Figure 11: Relative deviation from calculation for averaged points

Figure 12: Relative deviation of calculated curve and the one approximated experimental data
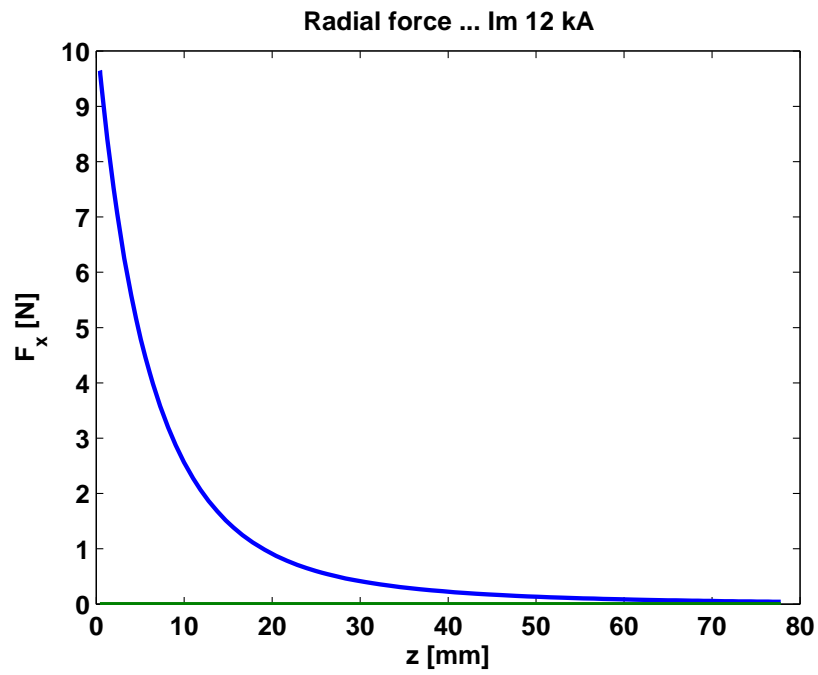

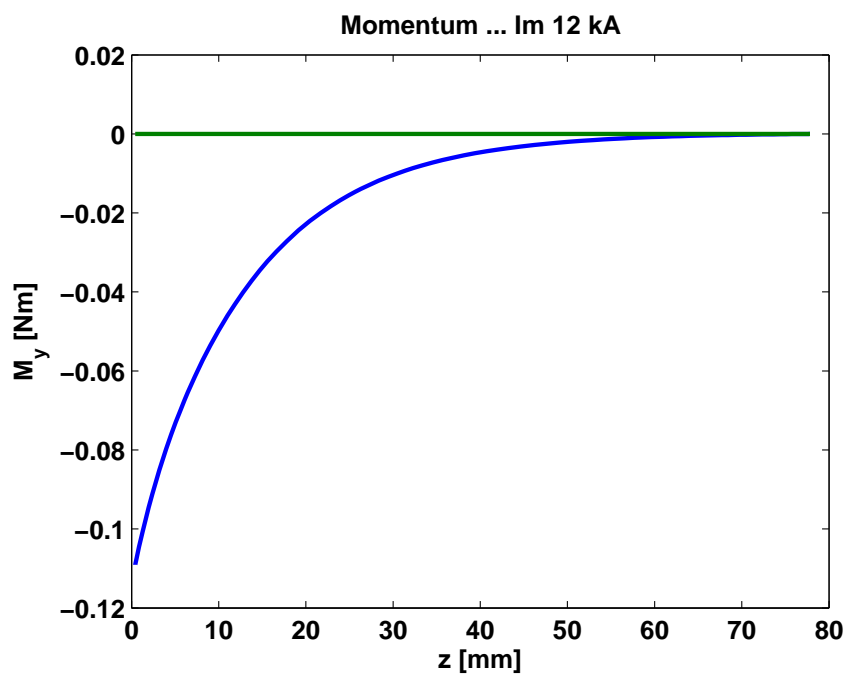Figure 13: Radial force for incorrect centering of 0.3 mm

Figure 14: Mechanical momentum for incorrect centering of 0.3 mm