SURFACE PROFILE EVALUATION BY FRACTAL DIMENSION AND STATISTIC TOOLS USING MATLAB

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Abstract

A lot of natural structures in industry applications can be hardly described by conventional methods – mostly statistic tools, because they are complex and irregular. A relatively new approach is the application of the fractal geometry that is successfully used in science, but an application in industry is sporadic and experimental only. However, the fractal geometry can be used as a useful tool for an explicit, objective and automatic description of production process data (laboratory, off-line and potential on-line).

The article is intended on application of the fractal geometry with combination of statistic tool for a quantification of metal surface changes of relatively new materials: iron aluminides in comparison with currently used chrome-nickel steels in contact with a glass melt. The software Matlab was used for the generation and classification of dividing lines (a surface profile or a surface roughness evaluation) and for evaluation of results.

Basic statistic and roughness parameters were used. The obtained data in a digital form can be described by the fractal geometry, which expressing the complexity degree of structured data (ideally) by means of a single number, the fractal dimension. The dimension can be estimated by many different methods. A compass method is one of them and the method is based on the measurement of the dividing line by different size of a ruler.

1   Introduction

Although continuously growing a competitive press to increasing quality of products activates a requirement of an objective measurement and control methods for materials, processes and productions, many structures (e.g. defects, surface, crack, time series from dynamic processes) can be hardly described by conventional methods, because they are complex and irregular. The fractal geometry [1-3] is a useful tool for analysis of complexities and irregularities that use fractal dimension (single number) for quantification; however an application in industry is sporadic and experimental only [4]. Currently, there are tools to monitor tree basic data format types: digitalized photos (application for the corrugation test of windows glass shields), time series (analysis for control systems) and topological one dimensional dividing lines (especially surface roughness). On account of this, we are developing three off-line programs and some are converted into monitoring systems.

The analysis of topological one dimensional dividing lines was developed for the quantification of metal surface changes of relatively new materials: iron aluminides in comparison with currently used chrome-nickel steels in contact with the glass melt a for laboratory use. The methodology was also used for an objectification of the corrugation test of glass sheet [5, in Czech]. The analysis was prepared in software Matlab that has suitable function for this reason.

2   Classification of Dividing Lines – Surface Profile Evaluation

Analyses were performed on samples of the iron aluminide Fe28Al4Cr0,1Ce and the chrome-nickel steel X15CrNiSi25 21 - EN 10095 (AISI 310) that were exposed to static and dynamic glass melt effects in different temperatures.

The methodology of the surface profile evaluation is shown in Fig. 1. Firstly, a digital camera takes a photo of a surface layer profile from a microscoped metallographic sample, Fig. 1, A.

Secondly, a dividing line is generated from the digital photography, Fig. 1, B, by the software tool developed in Matlab (using the command "contourc") that exactly defined the curve between material alloys and a surrounding - the dividing is obtained. The width of images is 2272 that matches 57,7 µm. Statistic tools and (or) the fractal dimension can describe the curve Fig.1, C, D.
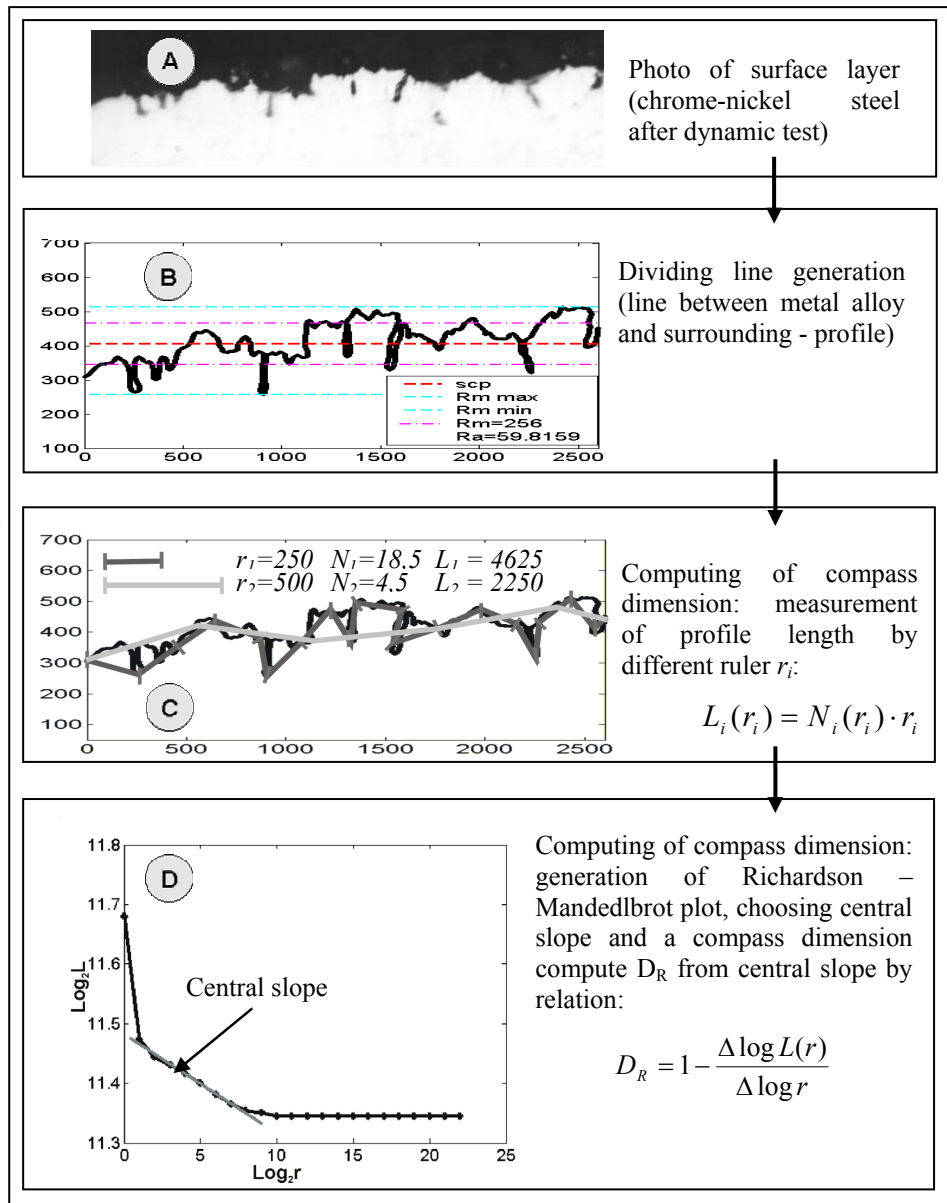

A — Photo of surface layer (chrome-nickel steel after dynamic test)

B — Dividing line generation (line between metal alloy and surrounding - profile)

C — Computing of compass dimension: measurement of profile length by different ruler r_i:

$$L_i(r_i) = N_i(r_i) \cdot r_i$$

$r_1=250$ $N_1=18.5$ $L_1 = 4625$
$r_2=500$ $N_2=4.5$ $L_2 = 2250$

D — Computing of compass dimension: generation of Richardson – Mandedlbrot plot, choosing central slope and a compass dimension compute D_R from central slope by relation:

$$D_R = 1 - \frac{\Delta \log L(r)}{\Delta \log r}$$

Central slope

Figure 1: Analysis of surface layer, dividing line generation from photography, evaluation by statistic and compass dimension

3   Fractal Dimension

The fractal dimension (also named the Hausdorff-Besicovitch dimension) is closely connected to fractals that were defined by Benoit Mandelbrot, though scientists found some geometric problems with specific objects before him (e.g. the measurement of coast lines per different length of rulers by Richardson) [1]. A potentially powerful property of the fractal dimension (FD) is describing complexity by using single number that defines and quantifies structures. The number is mostly a no integer value and FD is higher then the topological dimension. For example, the Koch curve (one of the most famous mathematical deterministic fractal, Fig. 2) has the topological dimension $D_T = 1$, but the FD $D_F = 1.2619$. A smooth curve as a line has the topological dimension $D_T = 1$ and the FD $D_F =$

1. The FD can be computed for set of points, curves, surfaces, topological 3D objects, etc. and if the FD is higher than the topological dimension, we name the objects fractals [1-3].
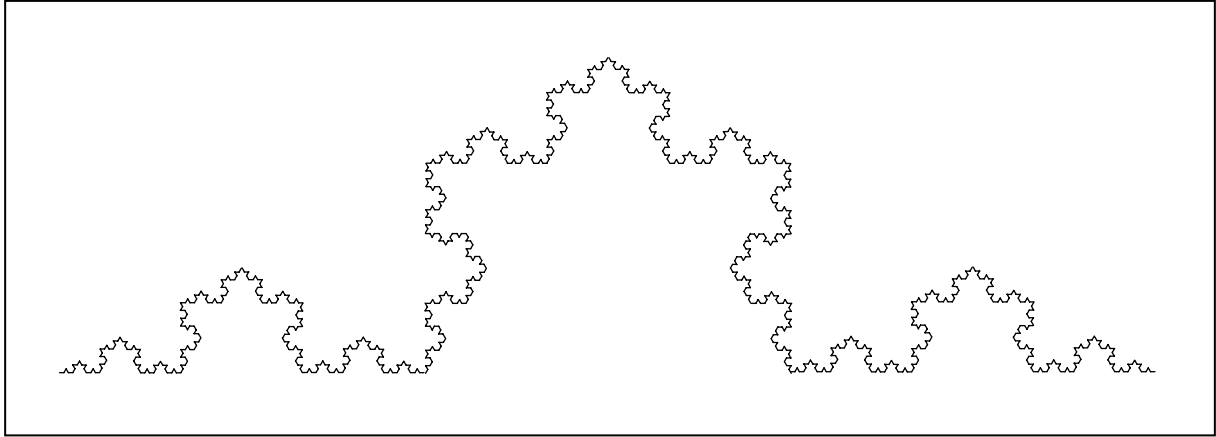

Figure 2: Koch curve

Analyses of data from production processes, quality controls, production tools, etc. should correspond with their characters and the computing the FD is suitable for highly structured data sets. Furthermore, the computation of the FD is explicit, objective and fast and it enables the application of the results for production control and quality monitoring.

The FD can be estimated by many different methods [1-3]. A compass method [2] is one of them and the method is based on the measurement of the dividing line (roughness profile) by different size of a ruler (Fig. 1, C) via the equation:

$$L_i(r_i) = N_i(r_i) \cdot r_i \qquad (1)$$

L_i is a length in i-step of the measurement, r_i is a ruler size and N_i is a number of steps needed for the measurement that is given by a power law:

$$N(r_i) = const. \cdot r_i^{-D_R} \qquad (2)$$

If the line is fractal and hence the FD is larger than the topological dimension, the measured length increases as the ruler size is reduced (Fig. 1, C). Using equations (1) and (2):

$$L_i(r_i) = N_i(r_i) \cdot r_i = const. r_i^{-D_R} \cdot r_i = const. \cdot r_i^{1-D_R} \qquad (3)$$

D_R is the compass dimension.

Logarithmic dependence between $\log_2 N(r_i)$ and $\log_2 r_i$ is called the Richardson-Mandelbrot plot (Fig. 1, D). The compass dimension is then determined from slope s of the regression line (Fig. 1, D):

$$D_R = 1 - s = 1 - \frac{\Delta \log_2 L(r)}{\Delta \log_2 r} . \qquad (4)$$

Although the typical dependence consists of three-parts slope, only central part (the central slope) is important for the compass dimension computing. The compass dimension D_R is multiplied by 1000 for better confrontation, $D_{R\,1000}$.

4   Statistic Tools

Basic statistic and roughness parameters were used such as: the Standard Deviation, the Range, the Average Surface Roughness (R_a), the Maximum Roughness (R_m or R_{max}), the Average Maximum Height of the Profile (R_z), the Peak Count (Peak Density, P_c), the Mean Spacing (Sm), ... (Fig. 1, C).

5   Fractal Dimension versus Statistics

A comparison of statistical tools and the FD is possible, but should be done with care. The FD gives added information about the character of describing data sets and to say that the FD is better than statistics and vice versa is impossible. Furthermore, the FD should not be used separately because the dimension does not give all the information about data set captures. Using added parameters (statistics, topology, spectral analysis, etc.) together with the dimension brings benefits and is recommended. A decisive number (a testing number) for production control or quality monitoring (for example) can be computed from obtained parameters (including FD) by weight coefficients. In his thesis [6] the author shows some other methods to use the "cooperation" between the FD and statistics and gives examples of their application.

6   Examples of Results

The examples are shown in Fig. 3. Six digital photos of every metal sample profile in different position were made and analysed. The presented results R_a, R_m and $D_{R\ 1000}$ are an average of six measurements on a tested sample and Fig. 3 shows only one example of the six dividing lines.

7   Conclusion

Although fractal and statistic results correlate in these examples of results (a profile with higher R_a and R_m has higher $D_{R\ 1000}$), the estimated fractal dimension (in this article the compass dimension) is information about structure, but R_a and R_m are statistic information. This information can correlate, but it is not rule. The compass dimension indicates complexity of profile, which can be used as added information to statistic or as a single profile specification. The estimated fractal dimension can also be used for others dividing lines types such as a surface roughness classification.

The results of our research (from application to digitalized pictures, time series or a dividing line) show that the fractal dimension is potentially a powerful tool for explicit, objective and automatic description and quantification of complex data [5-8]. The possibilities of successful applications in industry are believed to be large.
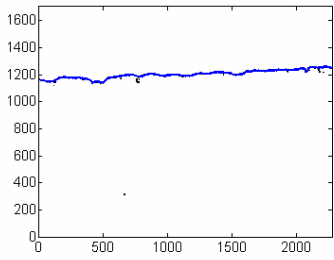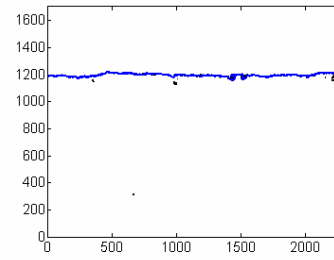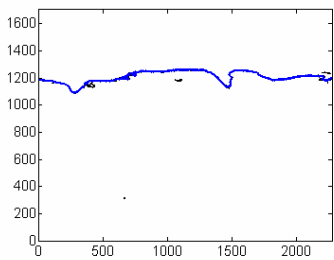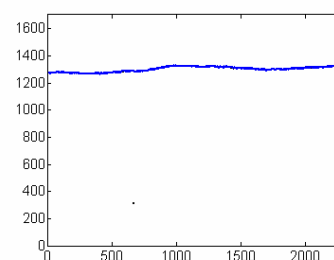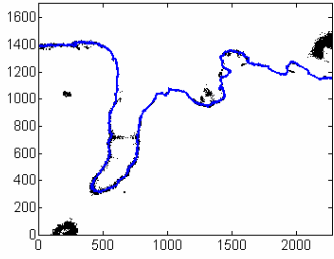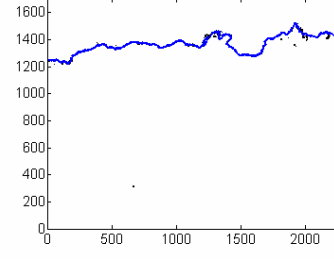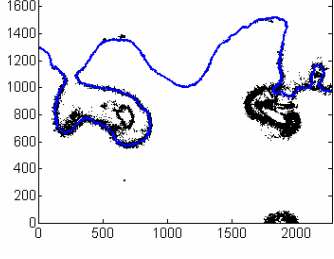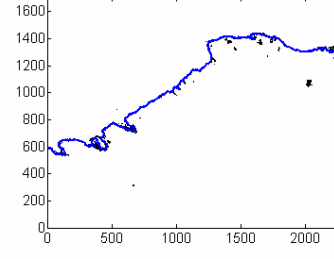
	Chrome-nickel steel EN 10095 (AISI 310)		FA - iron aluminide on base Fe$_3$Al	
	Dividing line - roughness profile (size in pixels)	R_m [μm] R_a [μm] $D_{o\,1000}$ [-]	R_m [μm] R_a [μm] $D_{o\,1000}$ [-]	Dividing line - roughness profile (size in pixels)
Ground state		3.07 0.79 1029.2	1.56 0.29 1055.8	
1100°C, 24 hour		4.28 1.01 1056.4	1.54 0.47 1012.0	
1250°C, 96 hour		28.11 8.59 1121.6	5.52 1.39 1100.3	
1350°C, 96 hour		24.08 6.09 1185.4	22.64 8.09 1124.0	

Figure 3: Examples of dividing lines chrome-nickel steel material and iron aluminide, after static glass melt effects in different temperatures and results of analyses

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