FINDING MECHANICAL PROPERTIES OF ISOTROPIC MATERIALS USING ERA IDENTIFICATION METHOD

R. Bartko, M. Maleková

KFIM Fakulta priemyselných technológií Púchov, TnUAD

Abstract

Elastic properties of aluminum (99,5%) and steel were computed from measurement of dynamic response of plates made from these materials. Natural frequencies and damping factors were identified using Eigensystem Realization Algorithm method. FEM model of plate was done and used for optimization process in Optimization Toolbox of MATLAB with aim to minimize difference between identified natural frequencies of plate and FEM model of plate. Elastic properties of aluminum and steel plates are found from properties optimized model as a result of optimization.

1 Introduction

Elastic properties of aluminum (99,5%) and steel plates were computed from measurement dynamic free response of plates made from these materials. The free initial state response of plate was measured and eigenfrequencies and modal damping rates were identified using time domain identification technique – ERA method. FEM model of plate was done and optimized in MATLAB with aim to minimize difference between identified natural frequencies of plate and FEM model of plate. Elastic properties of aluminum and steel plates are found from properties optimized model.

2 Identification Using Eigensystem Realization Algorithm Method

State variable equations of linear dynamical system are

$$\begin{aligned} \mathbf{x}_{k+1} &= \mathbf{A}\mathbf{x}_k + \mathbf{B}\mathbf{u}_k, & \mathbf{B} &= [\mathbf{b}_1 \quad \mathbf{b}_2 \quad \dots \quad \mathbf{b}_m] \\ \mathbf{y}_k &= \mathbf{C}\mathbf{x}_k, & \mathbf{C}^{\mathrm{T}} &= [\mathbf{c}_1^{\mathrm{T}} \quad \mathbf{c}_2^{\mathrm{T}} \quad \dots \quad \mathbf{c}_p^{\mathrm{T}}] \end{aligned} \tag{1}$$

where $\mathbf{x}$ is n-dimensional state vector, $\mathbf{u}$ an m-dimensional control input vector and $\mathbf{y}$ is p-dimensional measurement vector, $\mathbf{A}$ is $n \times n$ a transition matrix. The problem of eigensystem realization is as follows: given measurement functions $\mathbf{y}_k$, identified matrices $\mathbf{A}, \mathbf{B}, \mathbf{C}$. Free initial state response of system is given

$$\mathbf{Y}_k = \mathbf{C}\mathbf{A}^k [\mathbf{x}_1(0), \mathbf{x}_2(0), \dots, \mathbf{x}_m(0)] \tag{2}$$

A block data set Hankel matrix is constructed

$$\mathbf{H}_k = \begin{bmatrix} \mathbf{Y}_{k+1} & \mathbf{Y}_{k+2} & \cdots & \mathbf{Y}_{k+\eta+1} \\ \mathbf{Y}_{k+2} & \mathbf{Y}_{k+3} & \cdots & \mathbf{Y}_{k+\eta+2} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{Y}_{k+\xi+1} & \mathbf{Y}_{k+\xi+2} & \cdots & \mathbf{Y}_{k+\xi+\eta+1} \end{bmatrix} \tag{3}$$

The singular value decomposition of matrix $\mathbf{H}_0$ is found in the form

$$\mathbf{H}_0 = \mathbf{P}_H \mathbf{D}_H \mathbf{Q}_H^{\mathrm{T}} \tag{4}$$

where $\mathbf{P}_H, \mathbf{D}_H$ are orthogonal matrices and $\mathbf{D}_H$ is a diagonal matrix of singular values. The size of matrices can be truncated by ignoring “small” singular values and minimum realization is in the shape

$$\mathbf{A} = \mathbf{D}_n^{-1/2} \mathbf{P}_n^{\mathrm{T}} \mathbf{H}_1 \mathbf{Q}_n \mathbf{D}_n^{-1/2} \tag{5}$$

Singular value decomposition does not separate system and spurious modes and so remaining spurious modes have to be eliminated using another technique such as modal amplitude coherence.

The system frequencies and damping are found from the eigenvalues of matrix $\mathbf{A}$, which occur in complex conjugate pairs

$$\lambda_i = e^{\left(-\xi_i\omega_i \pm \omega_i\sqrt{1-\xi_i^2}\right)\Delta t} \tag{6}$$

where for the i-th mode λ is the eigenvalue, ξ is damping ratio, ω is natural frequency, Δt sampling interval.

3 Description of Experiment

Elastic properties of aluminium (99,5%) and steel plates were computed from dynamic measurements. The measurements and computational steps are summarized as follows:

a. Free initial state response of plate was measured using measuerement AD/DA card Humusoft MF 624

b. Eigenfrequencies and modal damping rates were identified using ERA method

c. FEM model of plate was done in MATLAB -

d. Elastic properties of aluminum and steel plates were found using Optimization toolbox in MATLAB

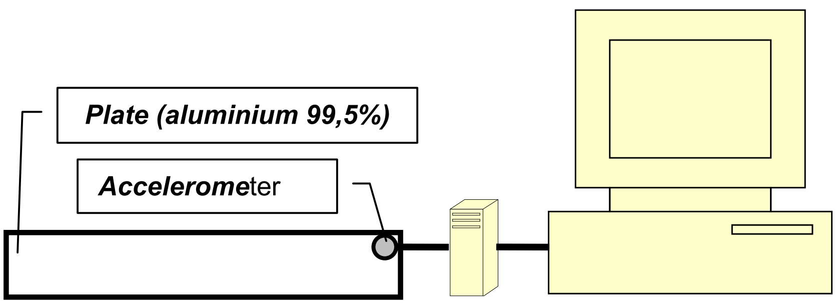

Figure 1: Schematic of measurement

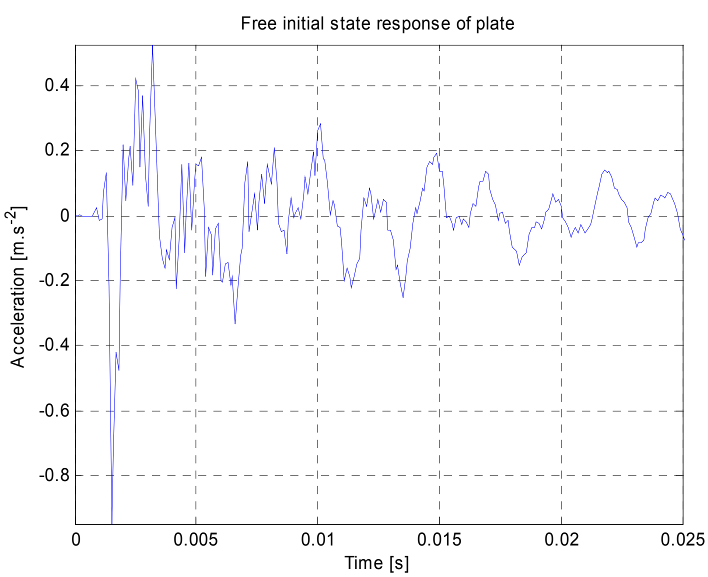

Figure 2: Free initial response of aluminum plate

Measurements were carried out on rectangular plate 32mm x 159mm x 1mm. Free initial state response of plate was measured with frequency 10kHz. Measurement was done in MATLAB with AD/DA card Humusoft MF 624 (Figure 1). Acceleration in one point (corner point) was measured. Example of one measured free initial state response of plate is shown on Figure 2. Power Spectral Density from 10 measurements is shown on Figure 3.

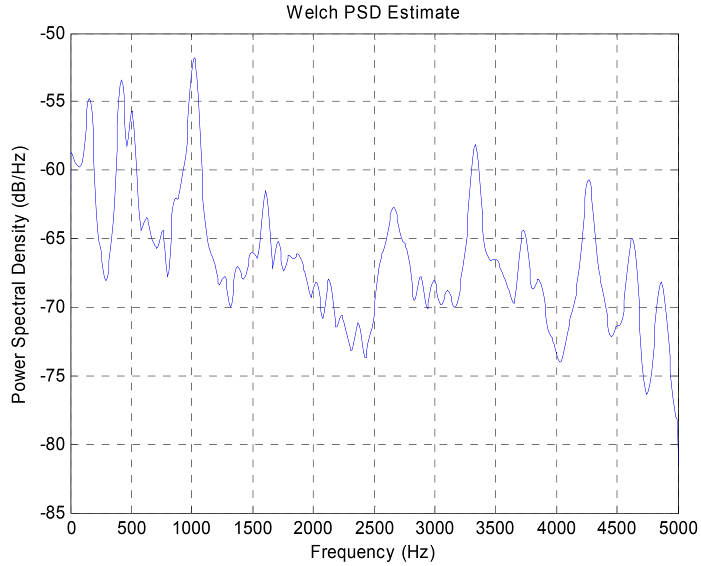

Figure 3: Power Spectral Density

4 Identification of Eigenfrequencies

Frequencies and damping ratios of plate were identified using ERA method. Results from one measurement are shown in Tab. 1. Identified frequencies were averaged. Sampling interval Δt was 0,0001s, number of rows of Hankel matrix was chosen 40. Identified frequencies obtained from ERA method were compared with damped eigenfrequencies obtained from Power Spectral Density (Figure 3).

TABLE 1: IDENTIFIED EIGENFREQUENCIES AND DAMPING FACTOR – ALUMINIUM PLATE

Number	Damping factor [%]	Frequency [Hz]
1.	8,41	159,74
2.	7,18	410,27
3.	8,27	496,42
4.	6,52	883,54
5.	1,39	1020,63
6.	1,82	1592,40
7.	1,36	2656,80
8.	0,72	3330,40

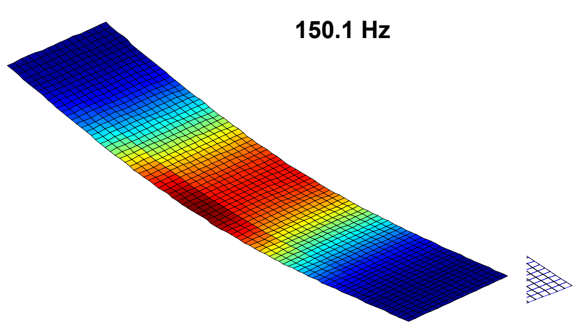

Figure 4: FEM model of plate and 1. bending shape - aluminum

5 Finite Element Model of Plate and Calculation of Elastic Properties Using Optimization in MATLAB

FEM model of plate used in this study is shown on Figure 4. Four node shells are used in model (1344 nodes and 1245 elements). Model has free ends. Material of model was isotropic with given density 2700kg/m^3 and unknown variables Young modulus E and Poisson's ratio μ.

MATLAB function `fgoalattain` was used to solve the goal attainment problem, which is one formulation for minimizing a multiobjective optimization problem. The function to be minimized was FEM model of plate. Vector of averaged identified eigenfrequencies was used as vector of values that the objectives attempt to attain. As variables of function were Young modulus E and Poisson's ratio μ. Results are in Table 2. and 3.

TABLE 2: ELASTIC PROPERTIES OF STEEL SAMPLES

Measured eigenfrequencies [Hz]	Calculated material properties	Eigefrequencies of optimized model [Hz]
74,3 500,0 648,5 1440,5 2006,4	E= $2{,}06.10^{11}$ Pa G= $7{,}72.10^{10}$ Pa μ= 0,299	78,4 495,6 652,8 1442,3 2008,9

TABLE 3: ELASTIC PROPERTIES OF ALUMINIUM SAMPLES

Measured eigenfrequencies [Hz]	Calculated material properties	Eigefrequencies of optimized model [Hz]
156,3 415,0 512,7 854,5 1025,0	E = 6,15 .1010 Pa G = 2,27 .1010 Pa μ = 0,353	150,1 406,2 509,5 863,3 1033,8

6 Conclusion and discussion

Elastic properties of aluminum (99,5%) and steel plates were computed from dynamic measurement of the free initial state response of plate. Only one accelerometer B&K was used. Problems were to excite all modes in interval from 0Hz to 5000Hz. Some modes (for example 3. mode for aluminum plate) were difficult to identify. Natural frequencies and damping factors were identified using ERA method and Power Spectral Density. Damping factor for metals is relatively lower, so damped and undamped eigefrequencies are almost the same. Damping factor for lower modes were identified using ERA method with higher damping ratio (for 1. mode was identified 8,4%), but damping factors were not used for calculation of elastic properties of materials. Finite element model of plate was done and optimized in MATLAB with aim to minimize difference between identified natural frequencies of plate and FEM model of plate. Elastic properties of aluminum plate were found from properties optimized model.

References

[1] COOPER, J.E., WRIGHT, J.R.: Journal of Guidance, Control, and Dynamics, Vol. 15, No. 2, 1992, 352.

[2] JUANG, J.N., PAPPA, R.S.: *Journal of Guidance, Control and Dynamics*, Vol. 8, No. 5, 1985, 620.

[3] JUANG, J.N., PAPPA, R.S.: *Proceedings of the AAS/AIAA Astrodynamics Specialists Conference, Univelt, San Diego,* 1985, 385.

Ing. Róbert Bartko, PhD., Ing. Monika Máleková
Katedra fyzikálneho inžinierstva materiálov, Fakulta priemyselných technológií, TnUAD
I. Krasku 491/30, 020 32 Púchov, e-mail: bartko@fpt.tnuni.sk, tel.: +421 42 4613840