ADVANCED ALGORITHMS FOR EQUALIZATION ON ADSL CHANNEL

T. Mazanec

Institute of Information Theory and Automation, Dept. of Signal Processing

Abstract

Digital subscriber line modems equalize response of line (channel) to fight with interferences and line dispersion which down the bit-rate of transmission. Main equalization principle is "Channel shortening" which is the base for time domain equalizers (TEQ). Many TEQ algorithms have been already proposed, mainly in theoretical sense. With growth of hardware performance, more complex methods can be applied in practice. Thus a proper simulations and fast implementations are necessary to achieve this.

We developed a toolbox which implements some simple and advanced time domain equalization algorithms and simulates ADSL transmission line with reference channels (loops). This paper introduces implementation details and comparison of selected equalization algorithms.

1 Introduction

Asymmetric digital subscriber line (ADSL) systems are wire-line based and mostly run on metallic cables which were designed for transmission of voice-band telephone signals. The crucial task of such system is restoration of degraded signal at receiver modem. Equalizers, adaptive digital filters, can do much of this work. The aim of this work is focused in time-domain equalizers particularly in advanced algorithms which could lead to higher bit-rates and or lower error rates.

Many TEQ algorithms have been already proposed, mainly in theoretical sense. With growth of hardware performance, more complex methods can be applied in practice. However before their use in ADSL box these algorithms has to be simulated and their performance vs. computational cost should be examined.

In this paper we present Matlab toolbox which implements selected TEQ algorithms and simulates ADSL transmission line. The paper is organized as follows: Section 2 briefly describes ADSL transmission system, Section 3 introduces time-domain equalization, gives a equalizer fundamentals and classification, Section 4 separately presents each TEQ algorithms in details than follow a section which shows some results and paper conclusion.

2 System description

Transmission on ADSL goes in brief: data are partitioned, modulated by discrete multitone modulation (DMT) and cyclic prefix is added. After transmitted signal pass through channel equalizer do its restoration. Cyclic prefix is removed and desired symbols are DMT demodulated.

Our implementation of ADSL downstream transmission line follows the ADSL transmission chain. Implemented transmitter maintain system setup, partitioning of input sequence, assembling of training sequence, discrete multitone modulation (DMT) and adding of cyclic prefix. Effect of passing the channel is simulated involving ADSL reference loops and custom noise models. Implemented receiver includes equalizer, prefix remover, DMT demodulator and QAM detector.

Bitloading behavior based on waterfilling principle [4] has been also included. With effort of more control over channel bitloading an adjustable masking of bitload was added as extension to regular waterfilling.

Transmission with channel equalizer basically operates in two different stages:

1. the training stage where the pseudo-random sequence is transmitted while equalizer adapts its filters at the receiver. Depending on the channel properties bitloading is also done here.
2. the transmission stage which maintain regular transmission of user data.

Both parts, equalizer update and user data transmission, are implemented in our toolbox. Presented performance measures in this paper are based on measurements in stage two, i.e. on the random user data sent through a channel.

3 Time domain equalization in ADSL

Main equalization principle in time domain, channel shortening, efforts the channel response in conjunction with equalizer response to be less distorting than channel response itself. Time-domain equalizers are based on different algorithms and so there are several different types of them.

Considering a maximum bit-rate through system as equalizer optimum there are complex and "non-smooth" mathematical relationships between optimum and channel (system) properties. We have analyzed and implemented TEQ based on mean square error optimization, per-tone channel properties and one blind optimization method. Algorithms based on mean square error (MSE) follow the system model shown on figure 1. These algorithms are trying to find optimal system delay (Δ), target impulse response (TIR) and equalizer coefficients (TEQ) to shorten system impulse response for given channel impulse response (CIR).

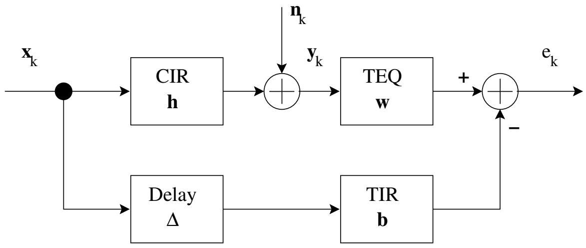

Figure 1: System model of TEQ with MSE optimization

These MMSE algorithms involve Unit Tap Constraint (UTC), Unit Energy Constraint (UEC) and theirs alternates. Computation of such algorithms is done by finding minimum of a MSE cost function with proper constraint to exclude trivial solutions. This practically leads to eigenvalue problem similar to (1).

$$\mathbf{Cw} = \lambda \mathbf{w} \ , \tag{1}$$

where $\mathbf{C}$ is composite matrix, $\mathbf{w}$ is vector of equalizer coefficients and λ is eigenvalue.

Several channel properties can be used for per-tone analysis. Optimizations based on these properties usually identify useful and useless parts of channel and minimize their ratio. Another optimization is driven by overall delay-spread of the channel.

Solution of these algorithms on per-tone basis can always yield to generalized eigenvalue problem [1]. Generalized Rayleigh quotient (2) fits well as an expression of such solution. Note that if one of composite matrices could form a constraint while the other involves the variables, described generalized eigenvalue leads to the standard eigenvalue decomposition.

$$\mathbf{w}_{opt} = \arg\min_{\mathbf{w}} \frac{\mathbf{w}^T\mathbf{A}\mathbf{w}}{\mathbf{w}^T\mathbf{B}\mathbf{w}} \ , \tag{2}$$

where $\mathbf{A}$, $\mathbf{B}$ are composite matrices and $\mathbf{w}$ is vector of equalizer coefficients. Structure of matrices $\mathbf{A}$, $\mathbf{B}$ depend on criteria of selected optimization method.

Algorithms covered by Rayleigh quotients are in particular: Maximum Shortening SNR (MSSNR), Minimum Inter-Symbol Interference (MinISI), Minimum Delay Spread (MDS) and Carrier Nulling Algorithm (CNA). Geometric SNR, mentioned above, leads to algorithms of Maximum Geometric SNR (MGSNR) and Maximum Bit-rate (MBR) which need multiple Rayleigh quotients to solve them. Note that multiple Rayleigh quotients problem, i.e. to find extreme of their product, requires non-linear optimization techniques.

4 TEQ algorithms and implementation details

An equalizer is viewed, in this paper, as usual adaptive filter and is trained by known (predefined) training symbol sequence. Optimization of equalizer could be done by iterating over delay (case of model 1) or by vector optimization for selected equalizer coefficients. The first mentioned is much simple to implement whereas the second needs to iterate across vector space and so Matlab *Optimization toolbox* was chosen to handle this. Initial conditions of filter coefficients are significant particularly for CNA and MDS algorithms. Multiple Rayleigh quotients based algorithms also need initialization but it has to be already adapted filter coefficients, possibly by a fast algorithm such UTC.

4.1 MSE based TEQ with unit tap constraint (UTC)

Algorithm iterates over the delay, Δ, and includes target response (TIR). Composite matrix $\mathbf{C}$ is assembled of transmitted and received signal correlation matrices and delayed crosscorrelation matrix. The most significant diagonal element of $\mathbf{C}$ gives the index i of unitary filter tap in each iteration. Enumerating MSE, defined by (3), for each iteration leads to the least MSE value and thereby to equalizer optimum.

$$MSE_{UTC} = \frac{1}{\mathbf{C}^{-T}(i,i)} \ , \tag{3}$$

where i is the index of unitary filter tap.

Optimal equalizer coefficients, $\mathbf{w}$, are consequently given by (4) with index i corresponding to optimal.

$$\begin{aligned} \mathbf{b} &= \frac{\mathbf{C}^{-T}(:,i)}{\mathbf{C}^{-T}(i,i)} \\ \mathbf{w}_{opt} &= \mathbf{R}_{yy}^{-1}\mathbf{R}_{xy/opt}^{T}\,\mathbf{b} \end{aligned} \tag{4}$$

where $\mathbf{b}$ is the vector of TIR response, $\mathbf{C}^{-T}(:,i)$ mean i-th column of $\mathbf{C}^{-T}$, $\mathbf{R}_{yy}$ is correlation matrix of received signal and $\mathbf{R}_{xy/opt}$ is crosscorrelation matrix optimally delayed.

4.2 MSE based TEQ with unit energy constraint (UEC)

Energy constrained MSE is another algorithm which iterates over the delay, Δ, and includes target response TIR. Since the energy constraint (5) involves properties of transmitted/received signals and not just unitary filter tap, eigenvalue decomposition has to be applied on composite matrix $\mathbf{C}$. Eigen-decomposition results in MSE value which corresponds to the least eigenvalue (6) and in target response determined by eigenvector of the least eigenvalue (7).

$$\mathbf{b}^T\mathbf{R}_{xx}\mathbf{b} = 1 \quad , \quad \mathbf{w}^T\mathbf{R}_{yy}\mathbf{w} = 1 \ , \tag{5}$$

where $\mathbf{R}_{xx}$ is correlation matrix of transmitted signal and $\mathbf{R}_{yy}$ is correlation matrix of received signal.

$$
\begin{aligned}
[\lambda, \mathbf{v}] &= \mathrm{EVD}\{\mathbf{C}\} \\
MSE &= \lambda_{min} && (6)\\
\mathbf{b} &= \mathbf{v}_{min} && (7)
\end{aligned}
$$

Minimal MSE value and corresponding TIR vector, $\mathbf{b}$, lead to optimal equalizer coefficients, $\mathbf{w}$, by equation (8).

$$\mathbf{w}_{opt} = \mathbf{R}_{yy}^{-1}\mathbf{R}_{xy/opt}^{T}\mathbf{b} \ , \tag{8}$$

4.3 SNR shortening (MSSNR) algorithm

Shortening SNR is driven by energy ratio of inner and outer part of windowed channel response. Algorithm iterates over delay and value of this delay determines the starting row of inner matrix, $\mathbf{H}_{win}$, which is extracted from convolution matrix $\mathbf{H}$ given by channel response, $\mathbf{h}$, and number of filter taps, N_w. The inner matrix, $\mathbf{H}_{win}$, has a dimension of $(\nu \times N_w)$ where ν is the length of cyclic prefix. Outer part consists of both $\mathbf{H}_{wall1,2}$ matrices (9).

$$\mathbf{H}^T = \Big[\underbrace{\mathbf{H}_{wall1}}_{delay}\underbrace{\mathbf{H}_{win}}_{\nu}\mathbf{H}_{wall2}\Big] \tag{9}$$

Composing energy matrices according to (10) and applying eigenvalue decomposition (11) over possible delays allows to find the maximum of energy given by each inner window part, i.e.: the highest of all eigenvalues determines optimal delay and corresponding eigenvector which gives desired equalizer coefficients (12).

$$
\begin{aligned}
\mathbf{A}^{-1/2} &= \mathrm{chol}\left\{\mathbf{H}_{wall}^{T}\mathbf{H}_{wall}\right\} \\
\mathbf{B} &= \mathbf{H}_{win}^{T}\mathbf{H}_{win} \\
\mathbf{C} &= (\mathbf{A}^{-1/2})^{-T}\mathbf{B}\,\mathbf{A}^{-1/2}
\end{aligned} \tag{10}
$$

$$[\lambda, \mathbf{v}] = \mathrm{EVD}\{\mathbf{C}\} \tag{11}$$

$$\mathbf{w}_{opt} = (\mathbf{A}^{-1/2})^{-1}\,\mathbf{v}_{opt} \tag{12}$$

4.4 Minimum delay spread (MDS)

Usage of cyclic prefix doesn't cover effects of whole channel response in ADSL. Delay spread was defined as a disturbing influence measure of system response (channel conj. equalizer). It follows proposed fact that further parts of system response, in time and related to CP, has more disturbing influence on system. With system response ($\mathbf{c} = \mathbf{h} \star \mathbf{w}$) equation (13) defines the delay spread, D, of the system. Iterating over equalizer coefficients, $\mathbf{w}$, and searching for the least delay spread, D, value leads to optimal equalizer design.

$$D = \sqrt{\frac{1}{E}\sum_{l=0}^{N+T-2}(l-\bar{l})^2\,|\mathbf{c}[l]|^2} \ , \tag{13}$$

where energy, E, and mean spread value, $\bar{l}$, are defined by (14)

$$E = \sum_{l=0}^{N+T-2} |\mathbf{c}[l]|^2 , \qquad \bar{l} = \frac{1}{E} \sum_{l=0}^{N+T-2} l \, |\mathbf{c}[l]|^2 \tag{14}$$

From the implementation point of view delay spread (13) is enumerated and Matlab function `fminsearch.m` is used to find the minimum. Function origins from *Optimization toolbox* and probes a vector space of the equalizer coefficients using *Simplex search* method. Note that eligible initial conditions ensures reliable convergence of algorithm.

4.5 Carrier nulling (CNA)

An equalizer based on CNA adapts itself to keep zero energy on unused tones which were transmitted with no information. This means that any energy received on unused tones comes only from channel interferences. The CNA method is obviously *blind*-method and note that there cannot be proved that CNA really leads to channel shortening [1]. Zeroing of unused carriers can be realized by finding minimum of cost function (15) which explores energy values of unused tones at receiver input $\mathbf{Y}$.

$$J = \sum_{i \in \mathcal{S}} \mathcal{E} \left[|\mathcal{F}_N[i,:] \mathbf{Y}^k \mathbf{w}|^2 \right] , \tag{15}$$

where $\mathcal{F}_N[i,:]$ represents i-th row of DFT matrix, $\mathcal{S}$ is a set of null-carriers and $\mathbf{Y}^k$ is matrix of received signal with dimension $N_w \times N_{FFT}$.

Minimum of cost function is given by enumerating within Simplex-searching method which is provided by `fminsearch.m`. Note that eligible initial conditions ensures reliable convergence of algorithm.

4.6 Maximum of Geometric SNR (MGSNR)

This algorithm exploits overall SNR measure (Geometric SNR) of the channel. The Geometric SNR involves signal to noise ratio on each used tone (SNR_i) and it's directly related to achievable bit-rate. With condition of rather higher channel SNR the Geometric SNR is given by (16).

$$\mathrm{GSNR}(\mathbf{w}) \approx \left(\prod_{i \in \mathcal{S}} \mathrm{SNR}_i \right)^{1/Nu} . \tag{16}$$

where $\mathcal{S}$ is a set of used tones and Nu is the number of used tones.

Author of [1] uses channel model (fig. 1) in spectral representation to express geometric SNR depending on filter coefficients (17).

$$\mathrm{GSNR}(\mathbf{w}, \mathbf{b}) \approx \sigma_x^2 \left(\prod_{i \in \mathcal{S}} \frac{|B_i|^2}{\sigma_{n,i}^2 \, |W_i|^2} \right)^{1/N_u} , \tag{17}$$

where $|B_i|^2$ and $|W_i|^2$ represent frequency response of filters, σ_x^2 is power of transmitted signal and $\sigma_{n,i}^2$ is power of noise for each tone.

The cost function (18), proposed by [1], allows to find an optimal equalizer design in mean of multiple Rayleigh quotients.

$$\begin{aligned} J(\mathbf{b}) &= \frac{1}{N_u} \sum_{i \in \mathcal{S}} \ln |B_i|^2 \\ &= \frac{1}{N_u} \sum_{i \in \mathcal{S}} \ln(\mathbf{b}^T \mathbf{Q}_i \mathbf{b}) \ , \end{aligned} \tag{18}$$

where $\mathbf{Q}_i = \mathcal{F}_N[i, 1:\nu+1]^H \mathcal{F}_N[i, 1:\nu+1]$ is composition of DFT matrix.

Initial vector of coefficients and MSE value as an initial constraint are obtained by other TEQ algorithm (e.g.: UTC). Maximum GSNR method itself uses "Constrained optimization" where the cost function (18) and the MSE constraint (19) are evaluated across TIR, $\mathbf{b}$, vector space and additional constraining conditions (20) are kept. MSE constraint indicates positive progress of optimization and the other constraint keeps solution non-trivial. The constrained optimization is provided by function `fmincon.m` of *Matlab Optimization toolbox*.

$$MSE = \mathbf{b}^T \mathbf{C}\, \mathbf{b} \ , \tag{19}$$

where $\mathbf{C}$ is a composite matrix of signal (cross-)correlations.

$$\begin{aligned} MSE &< MSE_{init} \\ \mathbf{b}^T \mathbf{b} &= 1 \end{aligned} \tag{20}$$

Optimal equalizer coefficients are obtained from optimal TIR coefficients by (21).

$$\mathbf{w}_{opt} = \mathbf{R}_{yy}^{-1} \mathbf{R}_{xy}^T\, \mathbf{b}_{opt} \ , \tag{21}$$

where $\mathbf{R}_{yy}$ is received signal correlation matrix and $\mathbf{R}_{xy}$ is crosscorrelation matrix.

5 Simulation results

All the equalizer algorithms, we had implemented, were tested within our toolbox. An example simulation results are presented on next figures. Figures 2a and 2b show results for reference channel loop "CSA #6". Figures 3a and 3b show results for reference channel loop "CSA #3".

Simulation conditions common for both reference loops were:

- Random user data was given by normally distributed random sequence which were partitioned to 500 of different data symbols.
- Noise added to channel reference was a custom noise model measured on concrete metallic cables. Model was provided by co-operative Department of Telecommunication Engineering, FEE CTU Prague.
- SNR of transmitted signal and added noise was set in range from 30 to 90dB.
- Each simulation for particular equalizer and SNR was repeated ten-times to reach some statistical independence.
- Bitloading was computed for each channel loop with an adjustable masking. Each carrier had the capability to carry up to 11 bits, i.e. up to 2048-QAM could be used on each carrier.

- Lower band of twenty carriers was left unused as ISDN/POTS band. Total number of usable carriers was then 235.
- Dimension of FFT in DMT (de)modulator was 512. Length of cyclic prefix was 40 samples long, i.e. each symbol was 552 samples long.
- Lengths of equalizer filter TEQ and target impulse response TIR were set to 32. System delay could vary from 3 to 39 samples.

Both figures on the left side (2a and 3a) show a bit-rates which were achieved by our customized bitloading algorithm for given channel loop. Figures on the right (2b and 3b) show the graphs of bit-error ratio established for varied SNR.

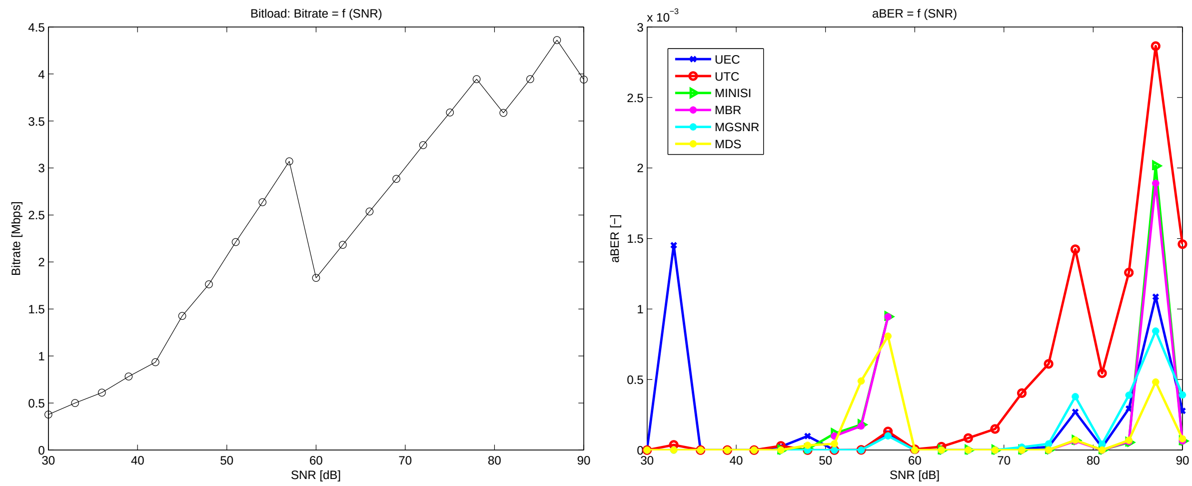

Figure 2: Example simulation results for CSA #6: a) achieved bit-rates for given SNR, b) average system bit-error ratios for given SNR

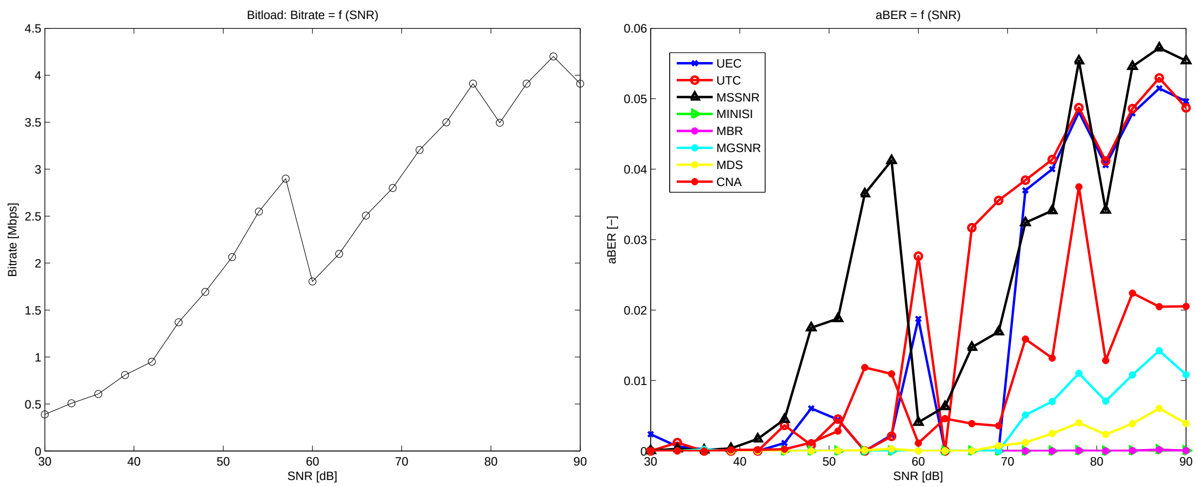

Figure 3: Example simulation results for CSA #3: a) achieved bit-rates for given SNR, b) average system bit-error ratios for given SNR

6 Conclusions

Presented toolbox is a part of the project which is pointed to broadband access networks. Nowadays the toolbox provides functional implementation of equalizers within ADSL transmission system and serve mostly for purposes of verification.

Bitloading algorithm has a significant influence on bit-error ratio (BER). Implemented bitloading is based on waterfilling principle, but waterfilling self has a trend to overcast so we had to adjust this behavior. This adjusting (reducing) of bitload with growing channel SNR is noticeable on figures 2a and 3a in “Simulation results” section. Reducing of bitload also cause some peaks on bit-error graphs (fig. 2b, 3b).

Simulation results shown in previous section were done within two different channel loops . Resulting error ratios reached by MSSNR and CNA methods are omitted in case of loop CSA #6, because theirs BER values were nearly erroneous in comparison with the rest of methods.

Channel loop CSA #3 has a lot of tappings opposite to CSA #6 loop which has none. We noticed, that some of used algorithms have significantly lower BER within tapped channels namely MSSNR and MinISI.

It has been emphasized that CNA is a blind optimization method and it doesn't lead to channel shortening. Despite of such properties it gives a satisfying results for tapped channels (see fig. 3b).

Acknowledgements

This work was supported by Academy of Sciences of the Czech Republic under project no. 1ET300750402.

References

[1] G. Ysebaert. Equalization and echo Cancellation in DMT-based Systems. SISTA-ESAT K.U. Leuven, Belgium, April 2004.

[2] J. A. C. Bingham. *ADSL, VDSL and Multicarrier Modulation.* A Wiley-Interscience Publication, John Wiley & Sons, Inc., New York, USA, 2000.

[3] J. M. Cioffi, N. M. W. Al-Dhahir. Efficiently Computed Reduced-Parameter Input-Aided MMSE Equalizers for ML Detection: A Unified Approach. *IEEE Trans. on Information Theory*, 42(3):903–915, May 1996.

[4] K. Van Acker. Equalization and Echo Cancellation for DMT Modems. SISTA-ESAT K.U. Leuven, Belgium, January 2001.

Tomáš Mazanec
ÚTIA AV ČR, P. O. Box 18, 182 08 Prague, Czech Republic, +420-2-6605 2472, mazanec@utia.cas.cz