NOISE REMOVAL BASED ON MAP ESTIMATOR IN THE WAVELET DOMAIN

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Abstract

This work deals with noise removal from an image data based on MAP estimator. Statistical model of the marginal probability density function (PDF) of digital images in the wavelet domain based on generalized Laplacian is used by this estimator. The model parameters was estimated by moment method. There has been presented powerful method for additive noise suppression. This method was also compared with other denoising algorithm based on suitable thresholding (Donoho-Johnston algorithm) of the wavelet coefficients.

1 Introduction

For many applications in image processing area, e.g. image reconstruction, denoising etc., it is useful to know the prior statistical model of images.

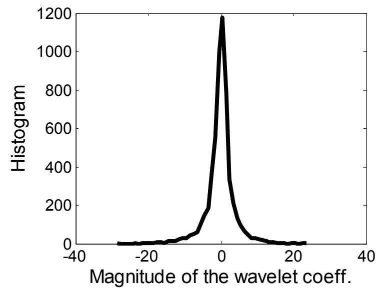


Figure 1: Histogram of the wavelet coefficients

Image can be represented by many linear or nonlinear transformations, e.g. Fourier, Karhunen-Loève or Wavelet Transformations. A dyadic decomposition as a form of The Discrete Wavelet Transform [1] was used in this paper. The Dyadic decomposition transforms an image data into the statistically independent wavelet coefficients. These wavelet coefficients are characterized by the marginal distribution, which is sharp peaked at zero with steep tails (in contrast to Gaussian distribution). It is intuitively evident that smooth area of an image data produces small wavelet coefficient, but edges, transitions etc. produce small number of the large wavelet coefficients. It is generally known that the value of the sample kurtosis κ of the Gaussian distribution is equal to 3. κ is defined as a fourth moment of X, divided by fourth power of its standard deviation σ. There is a histogram of the wavelet coefficients in the Fig. 1.

2 MAP estimator

When we consider signal x contaminated by additive noise n

$$y = x + n, \tag{1}$$

then the estimation of x, using MAP estimator [2], is given by

$$\hat{x}(y) = \arg\max_x p_{x|y}(x \mid y) = \arg\max_x p_{y|x}(y \mid x) \cdot p_x(x) = \arg\max_x p_n(y - x) \cdot p_x(x), \tag{2}$$

where p_n presents the probability density function of the noise, p_x denotes the prior probability density function of the signal (generalized Laplacian) and $p_{x|y}(x|y)$ stand for the posterior PDF . In this case was chosen Gaussian probality density function (3) for noise (consider aditive Gaussian noise with zero mean, $\mu = 0$) and the generalized Laplacian for the signal (4) in the wavelet domain

$$p_n(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}. \tag{3}$$

3 Model of the Probability Density Function

As mentioned in the first chapter, wavelet coefficients are characterized by interesting histogram, which is sharp peaked at zero with steep tails. The PDF can be modeled by generalized Laplacian (S. G. Mallat) [3], which is given by

$$p_x(x) \propto e^{-\left|\frac{x}{s}\right|^p}, \tag{4}$$

where parameter s controls the width of the distribution and parameter p controls the shape. These parameters should be estimated by moment method [2] (from second and fourth moment) or for example by least square error method. The estimation of parameters utilizing the least square error method minimizes the sum of the square of difference beetwen model of the probability density function and normalized histogram. This estimation method holds only for signal without additive noise. When signal is contaminated by additive noise then parameters can be estimated using moment method [2]. In our case were used the values of the parameteres, which were estimated from our image database, which contains a huge amounts of the scientific and multimidia images. Parameter p it takes value, for multimedia images, from 0.6 to 1.1.

4 Results

The implementation of the denoising algorithm can be seen in the Fig. 2, where *DWT* denotes the Discrete Wavelet Transform, *MAP est.* presents MAP estimator and *IDWT* stand for Inverse Discrete Transform. Four subbands are obtained in the first level of dyadic decomposition (DWT). This subbannds are called: LL1 – approximation, HL1 – vertical details, LH1 – horizontal details, HH1 – diagonal details.

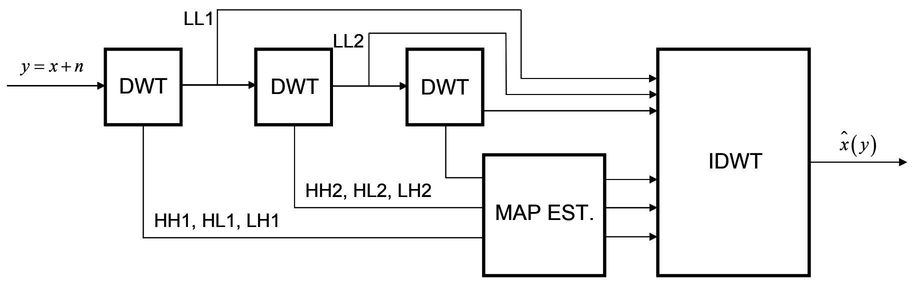
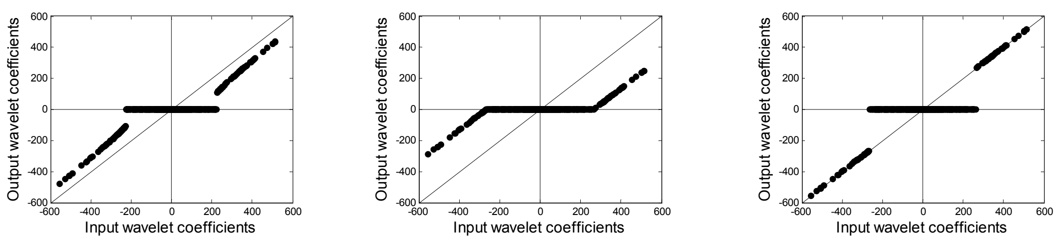


Figure 2: The implementation of the denoising algorithm

Below can be seen the function `BayesDen`, which represents implementation of the MAP estimator in accordance with equation (2)

```
function[band_den] = MAP_Den(band_noise,step,sigma,'method')
```

where `band_noise` denotes the wavelet coefficients of the noise signal, `step` presents sampling step of the PDF's, `'method'` can be `'nlse'` or `'moment'` and `sigma` is a standard deviation of the Gaussian additive noise.



a) b) c)

Figure 3: Transfer function, band HL3, a) MAP estimator, b) soft thresholding, c) hard thresholding

The transfer function of the MAP estimator and the transfer function of the soft and hard thresholding estimator shows the Fig. 3. There is a summary of obtained PSNR in the Fig. 4. PSNR for 8-bits images can be computed by

$$PSNR = 10 \cdot \log_{10}\left(\frac{255^2}{MSE}\right), \tag{5}$$

where *255* denotes the maximum pixel value in image and *MSE* means Mean Square Error.

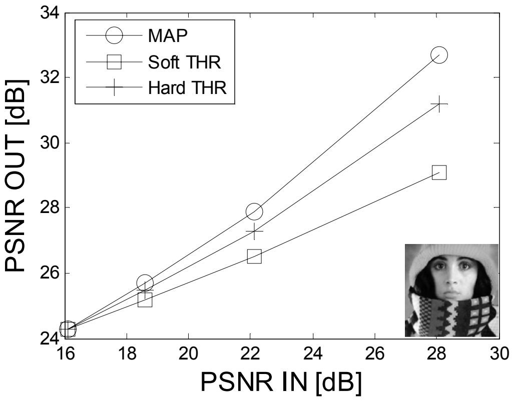


Figure 4: PSNR and testing image

5 Conclusion

As can be seen in the Fig. 4, in this experiment, the best results were obtained by MAP estimator. The denoising methods, which are based on Bayesian statistics are one of the most powerful methods for additive noise suppression. There has been presented MAP estimator, which uses statistical models (based on generalized Laplacian) of the wavelet coefficients like a prior description of the images. The further work will deal with modeling of the dark frame images.

6 Acknowledgement

This work has been supported by the grants No.102/05/2054 and No.102/03/H109 of the Grant Agency of the Czech Republic and by the research project MSM 6840770014 of the Ministry of Education, Youth and Sports of the Czech Republic. This work has been also supported by the grant CTU0610213 “Noise Removing from an Image Data Based on Bayesian statistics” of the CTU Prague.

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