SYNTÉZA KOMPENZÁTORU ENERGETICKÉ SÍTĚ POMOCÍ MATLAB OPTIMIZATION TOOLBOXU

SYNTHESIS OF A COMPENSATION OF THE ELECTRICAL NETWORK USING MATLAB OPTIMIZATION TOOLBOX

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Abstract

An algorithm is described for defining passive parameters of two-poles RLC, with whose help it is possible to compensate a three-phase load. The method is valid for sinusoidal or nonsinusoidal, balanced or unbalanced three-phase power system with linear or nonlinear load. The problem is formed as an optimization problem for minimizing losses in line. Given calculation method is illustrated by numerical problem.

Defining the solved problem

Three-phase non-linear load of inductive character is connected to balanced three-phase network, whose voltages are sinusoidal functions with period T, Fig. 1. The load draws currents $i_1(t)$, $i_2(t)$, $i_3(t)$ that are periodical, generally unbalanced and nonsinusoidal. To the load terminals shunt compensators are attached that contain two-poles RLC. The inductance of reactance coils is chosen so that resonance frequency f_r of the two-poles is distanced from the frequency of higher harmonics generated by the nonlinear load, usually f_r = 189 Hz or f_r = 134 Hz. The network is connected with the load through line with currents $i_{l1}(t)$, $i_{l2}(t)$, $i_{l3}(t)$. Time course of voltage on load terminal is known and we define parameters R, L, C of compensation two-poles, for which the losses in line are minimal. We minimize the functional, which is objective function

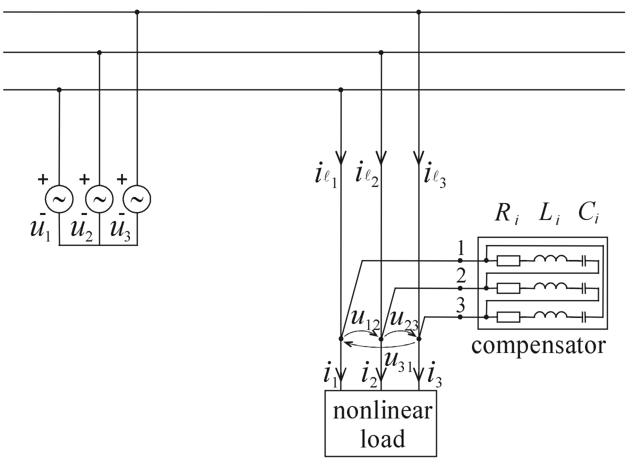


Fig. 1. Three-phase circuit structure.

$$F = \frac{1}{T}\int_0^T (i_{l1}^2 + i_{l2}^2 + i_{l3}^2)\,\mathrm{d}t \tag{1}$$

Calculation of compensation of two-poles parameters

Instantaneous values of phase voltages and line voltages of a balanced network are

$$\begin{aligned} u_1 &= U\sin\omega t \\ u_2 &= U\sin(\omega t - 2\pi/3) \\ u_2 &= U\sin(\omega t + 2\pi/3) \end{aligned} \tag{2}$$

$$\begin{aligned} u_{12} &= \sqrt{3}U\sin(\omega t + \pi/6) \\ u_{23} &= \sqrt{3}U\sin(\omega t - \pi/2) \\ u_{12} &= \sqrt{3}U\sin(\omega t + 5\pi/6) \end{aligned} \tag{3}$$

Currents in wye-connected compensation two-poles $R_i L_i C_i$ (i = 1,2,3) are

$$\begin{aligned} i_{12} &= I_{12}\sin(\omega t + \pi/6 - \psi_1) \\ i_{23} &= I_{23}\sin(\omega t - \pi/2 - \psi_2) \\ i_{31} &= I_{31}\sin(\omega t + 5\pi/6 - \psi_3) \end{aligned} \tag{4}$$

where $$I_{ij} = \sqrt{3}U\left[R_i + (\omega L_i - 1/\omega C_i\right]^{-\frac{1}{2}} \qquad i,j = 1,2,3;\ i \neq j \tag{5}$$

The reactance coil has inductance L_i, which is defined so that the two-pole has the chosen resonance frequency f_r, thus

$$L_i = \frac{1}{\omega_0^2 C_i}, \qquad \text{where} \quad \omega_0 = 2\pi f_r \tag{6}$$

Let its resistance be k-multiple of inductive reactance, thus

$$R_i = k\,\omega L_i = \frac{k\,\omega}{\omega_0^2 C_i} \tag{7}$$

Then phase angle $\psi_1 = \psi_2 = \psi_3 = \psi \in \langle 0, \pi/2 \rangle$, when

$$\tan\psi = \frac{1}{R_i}\left(\omega L_i - \frac{1}{\omega C_i}\right) = \frac{1}{k}\left(1 - \frac{\omega_0^2}{\omega^2}\right) \tag{8}$$

It is possible to express equation (5) using equations (6) and (7) in the form

$$I_{ij} = \frac{C_i U_{ij}}{A} \tag{9}$$

where

$$A^2 = \frac{\omega^2}{k^2 \omega_0^4} + \left(\frac{\omega}{\omega_0^2} - \frac{1}{\omega}\right)^2 \tag{10}$$

instantaneous line-currents in eq. (1) are calculated from equations

$$i_{l1} = i_1 + i_{12} - i_{31}, \quad i_{l2} = i_2 + i_{23} - i_{12}, \quad i_{l3} = i_3 + i_{31} - i_{23} \tag{11}$$

So the optimization problem is formulated. The solutions are the parameters of compensation two-poles. If the load is linear, unbalanced and of inductance character, it draws currents

$$\begin{gathered} i_1 = I_1 \sin(\omega t - \varphi_1), \quad i_2 = I_2 \sin(\omega t - \varphi_2 - 2\pi/3), \quad i_3 = I_3 \sin(\omega t - \varphi_3 + 2\pi/3) \\ \varphi_1, \varphi_2, \varphi_3 \geq 0 \end{gathered} \tag{12}$$

Numerical minimization of the objective function (1)

All above-mentioned formulas were implemented using programming language of computational system MATLAB and MATLAB Optimization Toolbox. At the beginning current amplitudes I_{12}, I_{23} and I_{31} had been computed – see equations (5). Constants definitions and auxiliary computations are not given here, as it is mentioned above. In the second step variables A_1, A_2 a A_3 were computed using equations (10), i.e. for example
`A1=(0.1.^2).*((omg.^2)./(omg0.^4))+(((omg./(omg0^2))-(1./omg)).^2);`
Computation followed with calculating of relevant amplitudes according to equation (5), i.e. for example `I12=C1.*(U12./sqrt(A1));` and currents in compensation two-poles using equation (4) , i.e. for example
`i12 = I12.*sin(omg.*t+(pi./6)-psi1);`
Before current in the load were calculated, we had computed following auxiliary variables, which represents final angles. We need to calculate these angles to make program code more transparent and we need to know it in the next part of computation, i.e. for example
`ang_i2 = omg.*t+((-52.*(2.*pi./360))-((2./3).*pi));`
Calculation of current in the load according to equation (12), i.e. for example
`i1=I1.*sin(ang_i1);`
The part of program code shown above generated a course of currents in case of linear load. In case of non-linear load this course must been adjusted, i.e. for example
`i1 = ~((mod(ang_i1,pi) < angle_4_t) & (mod(ang_i1,pi) > 0)).*i1;`
Some parts of the currents i_1, i_2 and i_3 courses had been levelled with the zero by this part of program code, according to value of variable *angle_4_t*. This method produced required course of currents. At the end of computation the courses of currents i_{l1}, i_{l2} and i_{l3} were calculated with help of conditions (11)

```
il1=i1+i12-i31;          il2=i2+i23-i12;          il3=i3+i31-i23;
```

Final sum of squares of these currents was computed: `y=(il1.^2)+(il2.^2)+(il3.^2);`

The numerical integration was based on equation (1). A standard MATLAB functions *quad* and *quadl* can be used. These functions used recursive adaptive Simpson quadrature algorithm. Function quad(*fun*, *a*, *b*) approximates the integral of function *fun* from *a* to *b* within an error of 10^{-6}. Function *fun* accepts vector *x* and returns vector *y*. Using form quad(*fun*, *a*, *b*, *tol*) uses an absolute error tolerance *tol* instead of the default (10^{-6}). In our calculations it was needed to set this tolerance usually between 10^{-7} and 10^{-9} to reach an adequate accuracy of integration. For this reason we used function *quadl* instead of *quad.* The function *quadl* should be more efficient with high accuracies and smooth integrands. Finally we used this function in the following form

```
quadl('fun',0,T,1e-8,[],C1,C2,C3) / T;
```

Result of this integration represents our objective function. To solve optimization problem, we applied standard MATLAB functions *fminsearch*, *fminunc* and *fmincon* included in MATLAB Optimization Toolbox.

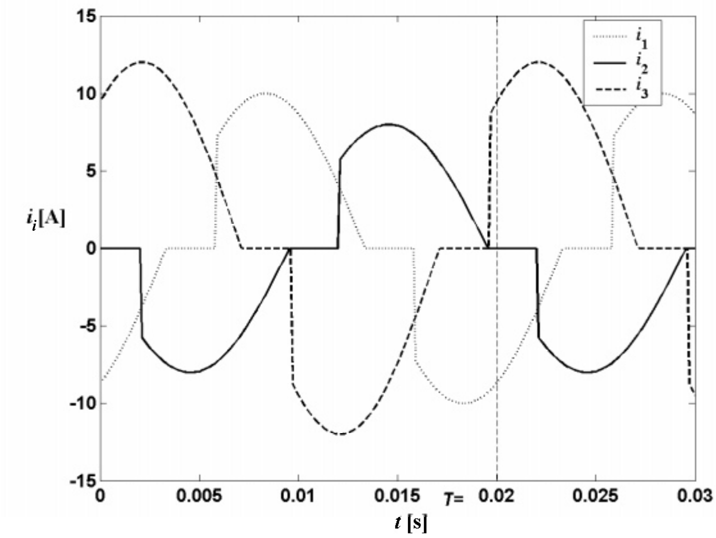
Function *fminsearch* is generally referred to as unconstrained non-linear optimization. We used it in form

```
[min,fval,exitflag,output]=fminsearch(@objective_f,input,options);
```

The variable *options* represent set of initial parameters of this function.

Function *fminsearch* uses algorithm based on the Nelder-Mead simplex direct search method. This is a method that does not use numerical or analytic gradients as in *fminunc* or *fmincon* (see below). When the solving problem is highly discontinuous, *fminsearch* may be more robust than *fminunc*. Function *fminunc* is generally referred to as unconstrained non-linear optimization of multivariable function. We used it in form

```
options.GradObj='on';
[min,fval,exitflg,output,grad,hessian]=fminunc(@objective_f,input,options);
```



The variable *options* represents set of initial parameters of this function as above. Many parameters are same as parameters of the function *fminsearch*. We used special parameter GradObj sets '*on*' – it means that user defines computation of gradient for the objective function.

Function *fminunc* uses algorithm based on the BFGS (Broyden, Fletcher, Goldfarb, Shanno) Quasi-Newton method with a mixed quadratic and cubic line search procedure (in case of medium-scale optimization). The DFP (Davidon, Fletcher, Powell) formula is used to approximate the inverse Hessian matrix. In case of Large-Scale Optimization an algorithm subspace trust region method based on the interior-reflective Newton method is used. Each iteration involves the approximate solution of a large linear system using the method of preconditioned conjugate gradients (PCG). When it is needed to eliminate improper values of variables (e.g. negative values of capacitance) we implement function *fmincon*. This function finds a minimum of a constrained non-linear multivariable function. We used it in form

```
mat_A=[-1,0,0;0,-1,0;0,0,-1]; vec_b=[0;0;0]; options.GradObj='on';
[min,fval,exitflag,output,lambda_v,grad_v,hessian_v]=fmincon(@criteria_f,
input,mat_A,vec_b,[],[],[],[],[],options);
```

Variable *mat_A* represents the matrix **A** of the coefficients of the linear inequality constraints and *vec_b* represents corresponding right side vector **b** (i.e. $\mathbf{A}\,\mathbf{x} \le \mathbf{b}$).

Function *fmincon* uses algorithm based on the Sequential Quadratic Programming (SQP) method (in case of medium-scale optimization). Quadratic Programming (QP) subproblem is solved at each iteration. An estimate of the Hessian of the Lagrangian is updated at each iteration using the BFGS formula (see *fminunc* above). A line search is performed using a merit function. The QP subproblem is solved using an active set strategy.

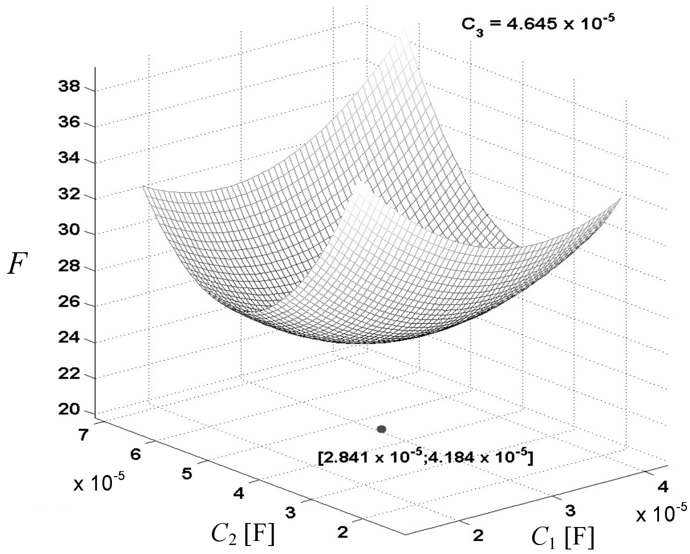


Fig. 3. Time-dependency of the current of the load.

Numerical problem

Network is according to Fig. 1. Line voltages are $\sqrt{3}U = 380\,\text{V}$, $\omega = 2\pi f = 100\pi$. Instantaneous values of currents in the load are (Fig. 2)

$$i_i = \begin{cases} 0 & \text{for } 0 < t < \alpha \\ I_i \sin(\omega t - \psi_i) & \text{for } \alpha < t < 2\pi,\ i = 1, 2, 3 \end{cases}$$

where $\psi_1 = -\varphi_1$, $\psi_2 = -\varphi_2 - 2\pi/3$, $\psi_3 = -\varphi_3 + 2\pi/3$. Calculation is done for $\alpha = 45^\circ$ and $I_1 = 10\,\text{A}$, $I_2 = 8\,\text{A}$, $I_3 = 12\,\text{A}$, $\varphi_1 = 60^\circ$, $\varphi_2 = 52^\circ, \varphi_3 = 68^\circ$.

Compensation is done using two-poles $R_i L_i C_i$ $(i = 1,2,3)$, for which $f_r = 189\,\text{Hz}$, $\omega_0 = 2\pi 189\,\text{s}^{-1}$, $k = 0{,}1$ is valid. Minimazing functional (1) we get

$C_1 = 2{,}841.10^{-5}\,\text{F}, C_2 = 4{,}184.10^{-5}\,\text{F}, C_3 = 4{,}645.10^{-5}\,\text{F}$

$L_1 = 0{,}1568\ \text{H}, L_2 = 0{,}1065\ \text{H}, L_3 = 0{,}0959\ \text{H}$

$R_1 = 4{,}926\ \Omega, R_2 = 3{,}345\ \Omega, R_3 = 3{,}013\ \Omega$

Conclusion

In this paper a method has been shown that enables to define the optimal values of parameters of compensation two-poles RLC, providing rigid supply mains. The proposed theory is valid for sinusoidal or nonsinusoidal, balanced or unbalanced three-phase power system. It can be easily extended to the power system with zero-sequence current, and/or voltages.

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