TRAJECTORY TRACKING FOR TAKAGI-SUGENO FUZZY SYSTEMS

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INTRODUCTION

In this work, we present an innovative technique for trajectory tracking of Takagi-Sugeno (TS) fuzzy models, besides the well-known scheme of Parallel Distributed Compensation (PDC). Among results on stabilization, input-output constraints and decay-rate specification, trajectory tracking has remained as a relatively unexplored topic in this field. In this work we not only established a new way to achieve trajectory tracking, but also we show simulation results for a two-link subactuated robotic manipulator.

TS FUZZY SYSTEMS

We start by defining the TS fuzzy model on which this work is based. Given a system

$$\dot{\mathbf{x}}_s = f(\mathbf{x}_s) + g(\mathbf{x}_s)u\,,\ \mathbf{x}_s \in \Re^n \qquad (1)$$

its TS fuzzy model is defined as follows:

$$\dot{\mathbf{x}} = \frac{\sum_{i=1}^{r} w_i(\mathbf{z})\{A_i\mathbf{x} + B_i u\}}{\sum_{i=1}^{r} w_i(\mathbf{z})} = \sum_{i=1}^{r} h_i(\mathbf{z})\{A_i\mathbf{x} + B_i u\}, \quad (2)$$

$$w_i(\mathbf{z}) = \prod_{j=1}^{p} V_{ij}(z_j)\,,\ h_i(\mathbf{z}) = w_i(\mathbf{z}) \Big/ \sum_{i=1}^{r} w_i(\mathbf{z})$$

where $\mathbf{x} \in \Re^n$ is the vector approximating $\mathbf{x}_s$, $\mathbf{z} \in \Re^m$ is the premise vector, the pair $\{A_i, B_i\}$ correspond to the i-th linearization of the system (1) and V_{ij} is the i,j-th membership function.

TRAJECTORY TRACKING

In order to perform trajectory tracking, we start by defining a linear system which generates the desired trajectory and is supposed to be of the same dimension of the TS fuzzy model (see [2]), i.e.:

$$\dot{\mathbf{x}}_d = A_d\mathbf{x}_d \qquad (3)$$

Now, our goal is to find a control law to guarantee that $\lim_{t\to\infty} \mathbf{e} = \mathbf{0}$, where the $\mathbf{e} = \mathbf{x} - \mathbf{x}_d$ is the tracking error vector. Taking its derivative, we have:

$$\dot{\mathbf{e}} = \dot{\mathbf{x}} - \dot{\mathbf{x}}_d = \sum_{i=1}^{r} h_i(\mathbf{z})\{A_i\mathbf{x} + B_i u\} - A_d\mathbf{x}_d$$

Notice that $\lim_{t\to\infty} \mathbf{e} = \mathbf{0}$ if $\dot{\mathbf{e}} = F\mathbf{e}$ where F is a stable matrix; so we can derive the desired control law just by solving u from the following equation:

$$\sum_{i=1}^{r} h_i(\mathbf{z})\{A_i\mathbf{x} + B_i u\} - A_d\mathbf{x}_d = F\mathbf{e}\text{, i.e.:}$$

$$u = \left(\sum_{i=1}^{r} h_i(\mathbf{z})B_i\right)^{\perp}\left(A_d\mathbf{x}_d + F\mathbf{e} - \sum_{i=1}^{r} h_i(\mathbf{z})A_i\mathbf{x}\right) \quad (4)$$

where $D^{\perp} = \left(D^T D\right)^{-1} D^T$ is the Moore-Penrose pseudo-inverse of matrix D. Note that u is a *nonlinear* control law.

SIMULATION RESULTS

We have considered a two-link subactuated robotic manipulator (see [3]), whose equations have been slightly modified in order to measure the involved angles according to Figure 1 (i.e., equilibrium point has been modified), while *sign* function has been approximated by a sigmoid, this is:

$$\begin{aligned}
\dot{x}_1 &= x_3 \\
\dot{x}_2 &= x_4 \\
\dot{x}_3 &= \frac{\left(M_{22}\left(u - F_1 - G_1 - C_{12}x_4 - C_{11}x_3\right) + M_{12}\left(C_{21}x_3 + G_2 + F_2\right)\right)}{\left(M_{11}M_{22} - M_{12}M_{21}\right)} \\
\dot{x}_4 &= \frac{\left(-M_{21}\left(u - F_1 - G_1 - C_{12}x_4 - C_{11}x_3\right) - M_{11}\left(C_{21}x_3 + G_2 + F_2\right)\right)}{\left(M_{11}M_{22} - M_{12}M_{21}\right)}
\end{aligned}$$

where:

$$M_{11} = 2.351 + 0.168\cos(x_2), M_{22} = 0.102$$
$$M_{12} = M_{21} = 0.102 + 0.084\cos(x_2)$$
$$C_{11} = -0.168\sin(x_2)x_4, C_{12} = -0.084\sin(x_2)x_4$$
$$C_{21} = 0.084\sin(x_2)x_3$$
$$G_1 = 9.81(3.921\sin(x_2) + 0.186\sin(x_1 + x_2 - \pi))$$
$$G_2 = 0.186\sin(x_1 + x_2 - \pi), F_1 = 2.288x_3 + 7.5sign(x_3)$$
$$F_2 = 0.175x_4 + 1.734sign(x_4), sign(x) = 2/\left(1 + e^{-10x}\right) - 1$$

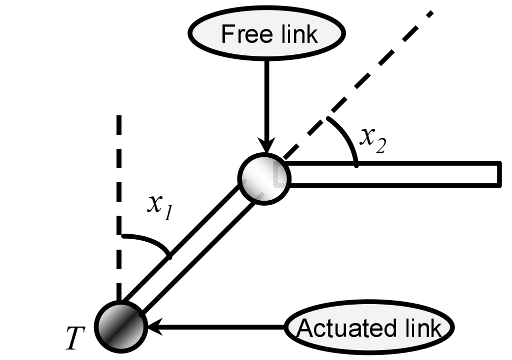

Figure 1: The PENDUBOT

The TS fuzzy model of the plant has been designed according to the equation (1), where $\mathbf{z} = [x_1 \quad x_3]$, i.e., the number of premise variables is $p = 2$. We use 4 rules ($r = 4$), covering the following ranges: $x_1 \in [-\pi/4, \pi/4]$ and $x_3 \in [-0.3, 0.3]$ (the states x_2 and x_4 are supposed to lie in complementary regions, see Fig. 2).

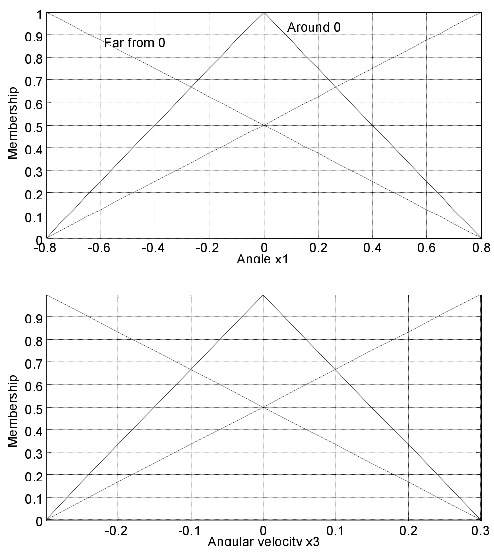

Figure 2: Membership functions

We have chosen a sinusoidal trajectory for x_1, which means that the whole system is supposed to oscillate around the unstable equilibrium point $x_1 = x_2 = 0$. To achieve this, we choose:

$$A_d = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}$$

and $F : \operatorname{Re} \lambda(F) = -3$. Simulation results are showed below. In Fig. 4 we can see the reference signal in bold line and system angle x_1 in dashed line. In Fig. 5 the control signal is displayed. We can see that reference signal tracking is succesfully achieved with a reasonable control signal.

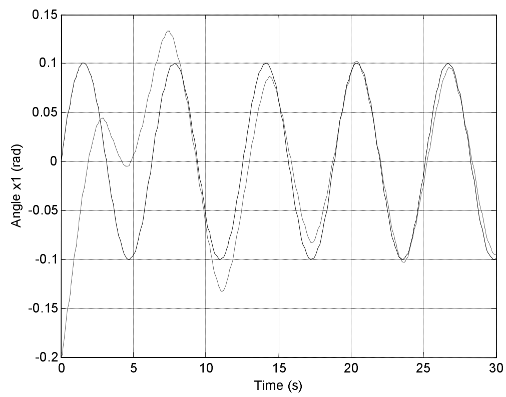

Figure 4: Trajectory tracking of angle x_1

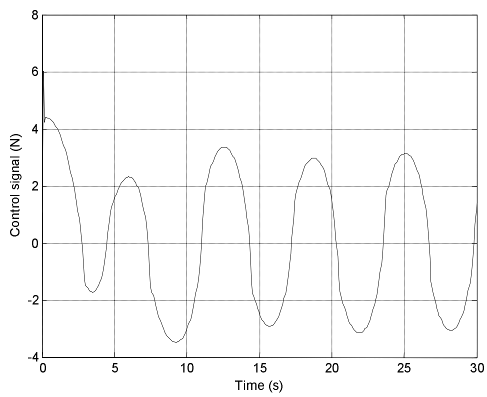

Figure 5: Control signal

CONCLUSION

PDC fuzzy control can solve difficult control problems with a suitable combination of accurancy and simplicity. Without losing this advantages, our approach achieve trajectory tracking of complex systems with a suitable modification of the PDC control law.

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