INTERVAL-SET ANALYSIS TOOLBOX APPLICATIONS for ESTIMATION and IDENTIFICATION PROBLEMS

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Computer Technology, Its Theoretical Foundations and Relation to New Present-Day Directions in Mathematics

The Interval-Set Analysis Toolbox (in a multidimensional linear space) is a form of implementation of the computer technology that is under its development at SRI NAS&NSA of Ukraine.

This computer technology is concerned with the new directions in mathematics: Interval Computations and Interval Analysis (http://www.cs.utep.edu/interval-comp/index.html)) [1,2], Set-Valued Analysis (http://www.kluweronline.com/issn/0927-6947) [3], Set-Theoretic Analysis (http://www.math.wvu.edu/~kcies/STA/STA.html)) [4].

The characteristic feature of the above-mentioned directions consists in the following: an investigation object here is not a concrete point, line or surface but a general-type set in a space. When problems are stated and solved, it is assumed that an initial information is specified as sets (intervals) and that a final computation algorithm implementation result is also a set. The approaches, developed within these directions, are widely used for data processing or estimation problems and when complicated systems are controlled. All this is done under non-stochastically stated uncertainty conditions. These problems emerge in many practical applications.

20 years already have passed from the date when the scientists of SRI NAS&NSA of Ukraine started developing the theoretic foundations of the interval-set analysis that find their application for control systems on the basis of set-theoretic uncertainty models. The set-theoretic approach implementation relies in this case on the following key assumption: a control plant (CP) parameter vector, a system state vector, uncontrollable disturbance and measurement error vectors know some a priori estimates that are represented as sets in their respective spaces. Such an a priori information is applied when adaptive control algorithms are synthesized. Set-theoretic identification processes run in these algorithms simultaneously with control processes, and obtained a posteriori parameter estimates are used when forthcoming controlling effects are calculated. The most important previous results are generalized in [5].

The further improvement in application of the set-theoretic approach to control theory problem solution is seen when the notion of an uncertain process is introduced [6] (in the later publications it is referred to as a chaotic process). This notion is based not only on the requirement that non-stochastic process values are to be limited at every time moment, but additional combined limitations are to be taken into account when they are imposed on these values at different time moments and form some set in a process phase space. If such combined limitations are used, it becomes possible to deal with some more rich and adequate information about an uncertain process. Yielded set-interval estimates for a state of a process itself and parameter estimates under identification are not so rough than in the case with simple process value limitation at every moment of time. The specific L-type is stressed in [6] for uncertain processes. The above-mentioned set-theoretic estimate in a process phase space is a polyhedron for combined linear limitations. The same estimate can be obtained as an uncertain process implementation processing result. When uncontrollable disturbances and observation errors act on a CP and are represented by the aforementioned model, it becomes possible to state and solve the parameter identification and CP state estimation [7] problem in a new way on the basis of the interval-set analysis.

Within the framework of the further development of the fundamental investigations to the advanced information technology stage technologies, the researchers of SRI NASU&NSAU started creating the new Interval-Set Analysis Toolbox in MATLAB. The Toolbox relies on the totality of the authors' MATLAB-based functions elaborated during the work on the project when it was executed and have got the grant of the Science and Technology Center in Ukraine (STCU) (http://www.stcu.kiev.ua) established by Ukraine, USA, Canada and European Union.

The initial version of this Toolbox was represented at HUMUSOFT s.r.o. Company (Prague, Czech Republic) and it was read at MATLAB 2001 Conference (Prague, 11.10.2001) (http://www.humusoft.cz/matlab01) [8]. As a result, the recommendations were obtained concerning the inclusion of the developed Toolbox into MATLAB Connections Directory, i.e. the worldwide collection of the commercial Toolboxes designed by the senior staff in different knowledge areas.

At present, the Interval-Set Analysis Toolbox is registered at The MathWorks, Inc. as the MATLAB Pre-Product Application within the framework of MATLAB Connection Program. The anticipated term for the commercial product availability is 2003. The Toolbox now keeps on being improved up to the commercial product, and the illustrating examples of its application in control systems are elaborated, in particular, in order to solve set-theoretic identification and filtration problems.

Some cases of how to apply Interval-Set Analysis Toolbox for the identification problem solution were already discussed in [9]. The present paper considers this matter further on.

The current Interval-Set Analysis Toolbox version functions and their categories and purposes are discussed below. The present Toolbox version is already essentially revised and reinforced in comparison with the version in [8].

Interval-Set Toolbox Function Categories and Its Purposes

The situation for today is that the initial Toolbox version contains more than 40 MATLAB-based functions. They mean the general solution within some linear inequality system set and the interval-set solution to a linear equation system with an interval uncertainty in a system matrix and in its right-side vector. According to the purpose, one may try to divide the main Interval-Set Analysis Toolbox functions into the following categories.

INITIAL Category (Table 1). This category includes different initialization functions in a finite-dimensional space of such sets as a hypercube, a hyperparallelepiped, a convex polyhedron, a half-space bounded by a hyperplane. A hypercube and a convex polyhedron are specified by the introduced special-purpose vertex and face matrices. They are described in such a way that is useful when one formalizes the procedure in accordance with which it is possible to find an intersection of a convex polyhedron and a hyperplane, while this intersection is the basis if a linear inequality system is solved in an interval-set manner. A hyperparallelepiped is specified by coordinates of a centre and by modules of admissible deviations of coordinates from a center. A hyperplane is specified by a direction cosine vector and by a scalar. The linear inequality system is converted into $\mathbf{CX} \leq \mathbf{B}$, where $\mathbf{C}$ is a specified matrix and $\mathbf{B}$ is a vector of respective dimensionalities.

INTERPOLYTOP Category (Table 2). It contains the functions with the help of which a hyperparallelepiped can be constructed. This hyperparallelepiped covers a set that is generated by initial system inequalities. The INTERPOLYTOP Category contains also the functions the purpose of which is to find an intersection of a priori specified convex polyhedron and a half-space and to construct a convex polyhedron. This procedure is applied in order to perform further multi-step clarification of a set-interval inequality system solution. An initial guaranteed interval estimation of a solution is a hyperparallelepiped.

Table 1. INITIAL Category. Set Initialization Functions in Finite-Dimensional Linear Space

INITIAL category	
vcub	Constructing a unit hypercube vertex matrix
permsun	Generating an array of only unique permutations of specified vector elements
Vcub_bit	Constructing a unit hypercube vertex matrix
gparal	Forming a hyperparallelepiped face matrix
V_G_CUB_FILE	Generating vertex and face matrices for a unit hypercube and writing them into the files
V_G_read_file	Reading vertex and face matrices for a unit hypercube from a files
V_G_disp_file	Displaying vertex or face matricesf faces of a unit hypercube
BOUND_PARAL	Inputting hyperparallelepiped boundaries and writing them into a file
vert_paral	Forming a vertex matrix for a hyperparallelepiped
Ineq_paral	Specifying hyperparallelepiped by a linear inequality system
Hyperplane	Generating a hyperplane
GENER_INEQ	Generating an linear inequalities system

Table 2. INTERPOLYTOP Category. Functions Constructing an Intersection of a Convex Polyhedron with a Half-Space

INTERPOLYTOP – category	
inter_polyhedron	Finding an intersection of a priori uncertainty polyhedron with a hyperplane
Fifunc1	Function used to establish belonging of a selected point to a halfspace
ind_G_eq_1	Establishing face matrix element indices that are equal to one
Adjac_ind	Finding numbers of polyhedron vertices adjacent to a selected vertex
New_vertices	Determination of new vertex coordinates and adequate conversion of vertex and face matrices
New_vert	Finding new vertex coordinates on the basis of considered current vertex and cut-off vertex
Cutoff_vertices	Forming an array of numbers for a priori polyhedron vertices, cut-off by a hyperplane
Remove_Vcept_next	Removing an index of a current cut-off vertex, passage to a next cut-off vertex
Add_ineq_nul_line	Forming an a posteriori linear inequality system and removing null-lines from a polyhedron face matrix
Ineq_inter	Finding an initial interval estimate for linear inequality system set solution

INEQUALITIES Category. It contains the following functions: to obtain a guaranteed interval estimate for a set solution to a linear inequality system; an equivalent conversion of an arbitrary inequality system to an inequality system in the first space orthante in order to obtain a guaranteed estimate of its solution; inverse conversion of a found set system solution in the first orthante into an initial finite-dimensional space.

EQUATIONS Category. It contains the functions aimed at initialization of a linear equation system with an interval uncertainty in a system matrix and in a right system side vector. The same category contains also the functions of reduction of such an equation system to a corresponding inequality system in each space orthante. The interval-set solutions to such inequality system can be obtained with the help of the aforesaid functions that belong to the INITIAL and INTERPOLYTOP directories.

CP Identification under Chaotic Disturbances. Problem Statement. Method Essence

The paper considers the control systems the mathematical model of which can be represented by the m-th order nonlinear differential equation

$$x_{n+1} = \varphi(X_n, U_n, L, n) + f_{n+1}; \quad (n = 0, 1, 2, \dots); \quad X_0 = X^{(0)}, \tag{1}$$

where $X_n^T = (x_n, x_{n-1}, x_{n-2}, \dots, x_{n-m+1}), \quad x_{-j+1} = x_j^{(0)} \quad (j = \overline{1, m})$ (2)

is an m-dimensional state vector; x_n is a scalar plant output; U_n is an r-dimensional control vector; L is an s-dimensional vector of unknown constant CP parameters; φ is a specified scalar function bounded on any limited set of its argument values; f_n is an external uncontrollable disturbance delivered to a CP output.

It is assumed that a disturbance $f_n \quad (n = 1, 2, ...)$ is a stationary L-type chaotic process that is defined in accordance with the following definitions given in [6].

Definition 1. *A stationary chaotic process with some connection interval* $[n, n+S-1]$ *(*$S > 0$ *is an integer) in a discrete time n is such a limited process* $f_n \quad (n = 1, 2, ...)$, *for which values of a state vector*

$$F_n^T = (f_n, f_{n+1}, \dots, f_{n+S-1}) \quad (n = 1, 2, \cdots) \tag{3}$$

belong to some bounded set $\mathbf{F}_S$ *in a space* E^S, *i.e.*

$$F_n \in \mathbf{F}_S \quad \forall n. \tag{4}$$

Definition 2. *A stationary chaotic L-type process is such a chaotic process* $f_n \quad (n = 1, 2, ...)$, *for which a set* $\mathbf{F}_S$ (4) *in a space* E^S *is specified by the restrictions*

$$p_{f1}(N) \le \frac{1}{N}\sum_{i=1}^{N} f_{n+i-1} \le p_{f2}(N); \quad N = \overline{1, \min(S, M)}; \quad n = \overline{1, M-N+1}, \tag{5}$$

where M is an observation width in a discrete time n; $p_{f1}(N), \quad p_{f2}(N)$ *are specified functions.*

A scalar observation y_n for a CP output value x_n is performed directly with an additive error z_n:

$$y_n = x_n + z_n, \tag{6}$$

and z_n is the stationary chaotic process that satisfies inequalities similar to inequalities (5) with specified functions $p_{zk}(N)$ instead of $p_{fk}(N) \quad (k = \overline{1, 2})$.

Two-sided inequalities (5), that are linear with respect to f_n and similar to z_n, specify some sets $\mathbf{F}$ and $\mathbf{Z}$ in respective spaces E^S. It is assumed that there are some initial estimates in the form of the sets $\mathbf{L}_0 \subset E^s$ and $\mathbf{X}_0 \subset E^m$. Therefore, an a priori estimation information is specified by the relations

$$L \in \mathbf{L}_0; \quad X \in \mathbf{X}_0; \quad f_n \in \mathbf{F}; \quad z_n \in \mathbf{Z}; \quad (n = \overline{1, M}). \tag{7}$$

The method used for CP identification on the interval-set analysis basis is as follows.

When proceeding from the results of the observations that satisfy expression (6), the sequence of set-theoretic parameter vector estimates

$$L \in \mathbf{L}_n, \quad \mathbf{L}_n \subset \mathbf{L}_{n-1} \quad (N = \overline{1, M}; \quad n = \overline{1, M-N+1}). \tag{8}$$

is made up on the trajectories of motion of system (1)-(2) under unknown values of controls U_n and in accordance with relations (7).

To describe the procedure, used to obtain the set-theoretic estimate $\mathbf{L}_n$ for a parameter vector L at an n-th step, the differential general-type set-evolution equation

$$\mathbf{L}_n = \mathbf{L}_{n-1} \cap \overline{\mathbf{L}}_n \quad \left(N = \overline{1, M}; \quad n = \overline{1, M - N + 1}\right), \tag{9}$$

may be used, in which $\overline{\mathrm{L}}_n$ is an a posteriori set-theoretic estimate obtained according to measurement result processing data at an n-th step.

In general, the identification of this type is the complicated problem because it needs solution to two systems of inequalities in the set terms. These inequalities comprise an introduced desired parameter vector and an unknown system state vector (including a vector as a whole and its separate components).

For the sake of simplicity, the paper considers further on the static CP class, the characteristic feature of which is the absence of the argument X_n in the function φ of expression (1).

Interval-Set Identification of a Static Linear CP under Stationary Chaotic Disturbances

The equation for a mathematical model of a static CP is

$$x_{n+1} = \varphi(U_n, L, n) + f_{n+1}; \quad (n = 0, 1, 2, \dots). \tag{10}$$

An inequality system, from which it is possible to obtain a set-theoretic parameter vector L estimate, follows from the joint consideration of observation equation (6) under $n = \overline{1, N}$, from inequality system (5) for f_n $\left(n = \overline{1, M}\right)$ and from a similar inequality system for measurement errors Z_n $\left(n = \overline{1, M}\right)$. In expression (6), substitute the expressions for x_n according to equation (10), change the indices ($n+1 \to n+i-1; \quad n \to n+i-2$) and perform the corresponding averagings in the left and right sides of the obtained equations. Then, in order to derive the final inequality system for estimation of L, the mentioned disturbance inequality systems are used.

Therefore, the following statement is true.

Statement. *A set-theoretic estimate* $\hat{\mathbf{L}}_M$ *for a parameter vector* L *of static CP* (10), *defined on the basis of measurement results M according to expression* (6) *in the presence of known values of controls* U_n, *is found when sets in a space* E^s *of parameters, specified by the inequalities*

$$p_1(N) \le \left(\frac{1}{N}\sum_{i=1}^{N} \varphi(U_{n+i-2}, L, n+i-2) - \bar{y}(n, N)\right) \le p_2(N); \quad \bar{y}(n,N) = \frac{1}{N}\sum_{i=1}^{N} y_{n+i-1}; \tag{11}$$

$$\left(N = \overline{1, M}; \quad n = \overline{1, M - N + 1}\right),$$

where $p_k(N) = p_{f\,k}(N) + p_{z\,k}(N) \quad (k = 1, 2)$,

with an initial space $\mathbf{L}_0 \subset E^s$.

For a static parameter-linear CP, the function $\varphi(U_n, L, n)$ in expression (10) takes the form

$$\varphi(U_n, L, n) = \sum_{j=1}^{s} l_i \;\varphi(U_n, n) = \Phi^T(U_n, n)\, L \quad (n = 0, 1, 2, \dots), \tag{12}$$

where $\Phi^T(U_n, n) = (\varphi_1(U_n, n), \varphi_2(U_n, n), \dots \varphi_s(U_n, n))$, and $\varphi_j(U_n, n)\left(j = \overline{1, s}\right)$ are specified functions.

For a parameter-linear CP, the set-theoretic estimate $\hat{\mathbf{L}}_M$ meets the system of the following inequalities that make up a polyhedron:

$$p_1(N) \le \left(A^T(n, N)\, L - \bar{y}(n, N)\right) \le p_2(N); \quad \bar{y}(n,N) = \frac{1}{N}\sum_{i=1}^{N} y_{n+i-1};$$

$$A^T(n,N)=[a_1(n,N),\dots a_s(n,N)];\quad a_j(n,N)=\frac{1}{N}\sum_{i=1}^{N}\varphi_j(U_{n+i-2},n+i-2)\quad (j=\overline{1,s}); \tag{13}$$

$$\left(N=\overline{1,M};\quad n=\overline{1,M-N+1}\right).$$

If an initial set-theoretic estimate $\mathbf{L_0}$ for a parameter vector L is a convex polyhedron, the estimate $\hat{\mathbf{L}}_M$ is also a convex polyhedron.

In the case when a CP is linear with respect to parameters and relative to a control vector (expression (10)), the following equation is derived for its mathematical model:

$$\varphi(U_n,L,n)=U_n^T\,L\Rightarrow\quad x_{n+1}=\sum_{j=1}^{s}u_{j\,n}\,l_j+f_{n+1}\qquad \left(n=\overline{1,M}\right). \tag{14}$$

For a CP of expression (14), system (13) contains M groups of two-sided inequalities. There are $\left(n=\overline{1,M-N+1}\right)$ inequalities in each N-th group. According to expression (14), system (13) can be unfolded as follows:

$$
\begin{aligned}
&p_1(1)\le\left(U_0^{\,T}\,L-y_1\right)\le p_2(1);\\
&p_1(1)\le\left(U_1^{\,T}\,L-y_2\right)\le p_2(1);\qquad (N=1)\\
&\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\\
&p_1(1)\le\left(U_{M-1}^{\,T}\,L-y_M\right)\le p_2(1);\\
&p_1(2)\le\left(\frac{1}{2}\left(\sum_{i=1}^{2}U_{1+i-2}\right)^T L-\frac{1}{2}\sum_{i=1}^{2}y_{1+i-1}\right)\le p_2(2);\\
&p_1(2)\le\left(\frac{1}{2}\left(\sum_{i=1}^{2}U_{2+i-2}\right)^T L-\frac{1}{2}\sum_{i=1}^{2}y_{2+i-1}\right)\le p_2(2);\qquad (N=2)\\
&\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\\
&p_1(2)\le\left(\frac{1}{2}\left(\sum_{i=1}^{2}U_{M-1+i-2}\right)^T L-\frac{1}{2}\sum_{i=1}^{2}y_{M-1+i-1}\right)\le p_2(2);\\
&-\;-\;-\;-\;-\;-\;-\;-\;-\;-\;-\;-\;-\;-\;-\;-\;-\;-\;-\;-\\
&-\;-\;-\;-\;-\;-\;-\;-\;-\;-\;-\;-\;-\;-\;-\;-\;-\;-\;-\;-\\
&-\;-\;-\;-\;-\;-\;-\;-\;-\;-\;-\;-\;-\;-\;-\;-\;-\;-\;-\;-\\
&p_1(M)\le\left(\frac{1}{M}\left(\sum_{i=1}^{M}U_{1+i-2}\right)^T L-\frac{1}{M}\sum_{i=1}^{M}y_{1+i-1}\right)\le p_2(N);\qquad (N=M).
\end{aligned}
\tag{15}
$$

System (13) is now written in the form of expressions (15), and it is seen that both of them contain $(M+1)M/2$ two-sided inequalities, and the s-dimensional vector L is present in them.

For the computer-aided identification process modeling, system (15) is transformed to the equivalent one-sided inequality system, and the number of the inequalities is doubled:

$$
\begin{pmatrix}
-\dfrac{1}{N}\left(\displaystyle\sum_{i=1}^{N}U_{n+i-2}\right)^T\\
\dfrac{1}{N}\left(\displaystyle\sum_{i=1}^{N}U_{n+i-2}\right)^T
\end{pmatrix}
L\;\le\;
\begin{pmatrix}
-p_1(N)-\dfrac{1}{N}\displaystyle\sum_{i=1}^{N}y_{n+i-1}\\
p_2(N)+\dfrac{1}{N}\displaystyle\sum_{i=1}^{N}y_{n+i-1}
\end{pmatrix},
\tag{16}
$$

$$\left(n=\overline{1,M-N+1};\quad N=\overline{1,M}\right).$$

The right-side matrix and left-side vector dimensionalities in system (16) follow from two-sided inequality system (15), while system (15) is equivalent to system (16). System (16) includes $(M+1)M$ inequalities.

Computer Modeling of a Static CP under Different Chaotic Disturbance Classes

Various scalar chaotic disturbance classes f_n (random; white, brown, pink noise; aviation noise; Lorenz attractor) were estimated, and the functions $p_1(N)$ and $p_2(N)$ with their majorants were estimated with respect to each class. The disturbance time series data were recorded into the arrays of up to 96000 elements.

The time disturbance series values are centred and standardized. Then, for each disturbance class, the independent selections are chosen from the standardized data array (1-96000 elements with the period of 1500 elements), and the length of the selections is 1000 elements with the forthcoming skip of 500 elements at each period. Therefore, it is possible to derive an ensemble of 64 selections each of which is 1000 elements long. The values of the functions $p_1^k(N)$ and $p_2^k(N)$, where $N=\overline{1,300}$, $k=\overline{1,64}$, are calculated on this ensemble and the boundary values $p_1(N)=\inf\limits_{k\in[1;64]}(p_1^k(N))$ and $p_2(N)=\sup\limits_{k\in[1;64]}(p_2^k(N))$ are found. The functions $p_1(N)$ and $p_2(N)$ are majorated by monotone increasing and monotone decreasing curves that pass, respectively, through local minima for $p_1(N)$ and through local maxima for $p_2(N)$. These overgraphs are plotted according to the following algorithm.

For the function $p_2(N)$, the majorant $p_{2m}(N)$ must monotonously decrease. Therefore, if $p_2(n+1)\le p_2(n)$, then these values are stored in the $p_{2m}(N)$ majorant value array. Otherwise, there is $p_2(n+1)\ge p_2(n)$, and one must go so many s steps back in the array of the $p_2(N)$ function value array that the decreasing condition $p_2(n+1)\le p_2(n-s)$ is met. The points $p_2(n-s)$ and $p_2(n+1)$ are connected by the straight line. In the band that has the width of $[(n-s+1);n]$, the values of the overgraph for $p_2(N)$ are calculated as the corresponding ordinates on this line. When the lower overgraph for $p_{1m}(N)$ is plotted for the function $p_1(N)$, everything is performed by analogy.

Figure 1 depicts the process when a part of the upper overgraph for $p_{2m}(N)$ is plotted for the white noise disturbance. Figure 2 shows the graphs of the majorants $p_{1m}(N)$ and $p_{2m}(N)$ of $p_1(N)$ and $p_2(N)$ for the white noise. The graphs of the majorants $p_{1m}(N)$ and $p_{2m}(N)$ for the aviation noise (a noise in the AN-70 airplane cockpit) and the Lorenz attractor disturbances are given, respectively, in Figures 3 and 4.

When linear inequality system is solved in the set-based way, then the values of the majorants $p_{1m}(N)$ and $p_{2m}(N)$ are taken instead of the values of the functions $p_1(N)$ and $p_2(N)$.

When a CP of expression (14) is modeled in a computer-aided manner, the real value $\widetilde{L}$ of an estimated parameter vector L is chosen as one of the uniformly distributed random vector values, and each vector element is positive and satisfies a certain two-sided inequality. For instance, $0.9\le|l_i|\le 3.6$ $(i=\overline{1,s})$ is proposed, and the real parameter vector value $\widetilde{L}$ is

$$\widetilde{L}^T=[1.422 \quad 2.742 \quad 1.717]. \tag{17}$$

under $s=3$.

The discrete time control vector series for 100 observation is derived by means of the function that generates uniformly distributed random elements of a (100xs)-dimensional array, where s is a vector dimension. Proceeding from the peak **U**-array element values and from the value of $\widetilde{L}$, the scale of the useful component $U_n^T\,\widetilde{L}$ $(n=\overline{1,100})$ for a CP output process x_{n+1} of expression (14) under $f_{n+1}=0$ is estimated.

If a CP is modeled, the time series of the aforesaid chaotic disturbance classes are generated with the scale that is usually equal to ¼ of the one for $U_n^T \widetilde{L}$. However, when the abilities of the proposed identification method were investigated with the special purpose provided that a disturbance level is higher than a useful component level, the "disturbance / signal" relation was taken as the one equal to 1.5. In the latter case, the interval-set identification process produced the set-theoretic parameter estimations that were, naturally, broader than in the case with small disturbances. But, if an interval estimate is derived in accordance with a found set-theoretic estimation, then an interval estimate centre approximated a real parameter vector value in as good way as it was seen under small disturbances.

The time series of the output CP process x_{n+1} are calculated on the basis of expression (14) as the sum of the values of $U_n^T \widetilde{L}$ and of the disturbances f_{n+1}.

Implementing the Algorithm Used for Set-Interval Identification of a Linear Static CP. Analysis of the Results

The interval-set identification procedure is implemented for the computer-modeled static linear CP in the presence of different chaotic disturbance classes. The solution to the identification problem is reduced to derivation of linear inequality system (15)-(16) pursuant to the CP output observation data under specified control values and estimated disturbance characteristics. The same solution is also reduced to the solution to the system with the use of the Interval-Set Analysis Toolbox functions.

When system (16) is solved in the interval-set way, the important point here is the derived initial guaranteed solution estimate. This aspect needs separate examination and consideration in some other publication. The present paper discusses the algorithms with the help of which it is possible to estimate an initial solution approximation, and these algorithms are dealt here with only schematically.

If there are small space dimensionalities (3-5), then it is possible to use the modified direct steps of Gaussian method in most cases together with a sequential re-numeration of variables. When this action is performed, then guaranteed interval estimates are obtained for all the variables. The set for an initial inequality system solution is a hyperparallelepiped.

If there are arbitrary space dimensionalities, the other method can be used and its basis is as follows: an initial two-sided inequality system is true in the whole space, reduce it by way of changing of variables to the first space orthant. All the new variables are non-negative in the first orthant. First of all, it is possible to obtain a guaranteed interval estimate as a hyperparallelepiped for a portion of new system variables, a number of which is equal to a space dimensionality. Choose these variables, and this choice defines a non-degenerated matrix of transition from a space of old variables into a space of new ones. Remove inequalities in accordance with which a guaranteed estimate is found, and a remaining initial inequality system part is transformed with the use of the matrix of transition into a new space of variables. A set-theoretic system solution is further on sought for in a space of new variables. A true solution, found as a polyhedron, is transformed into a space of initial variables.

An initial guaranteed interval estimation is found, and then inequalities of an initial system (or of its part transformed into a space of new variables) are permuted. After such a permutation, the first system inequalities become the ones that cut maximum parts in minima decreasing order off from an initial solution hyperparallelepiped-form estimate. When implementing the process of sequential intersections of half-spaces (formed by each permuted inequality) with a polyhedron (remaining after an initial estimate), then there is the essentially greater number of inequalities that turn out to be not information-carrying. Non-informativity of a respective inequality is characterized by the fact that a half-space, formed by it, does not intersect a remaining polyhedron. Therefore, a real set-theoretic solution to a system can be found much faster by means of the present permutation procedure.

Then, a set-theoretic solution to system (16)-(15) is found by means of the proposed identification method as the result of a sequential intersection of half-spaces, that are specified by each inequality, with an initial hyperparallelepiped-form solution estimate. If all the possible intersections are found when not information-carrying inequalities are removed, a real set-theoretic system solution is obtained, and this solution is a set-theoretic estimate for a parameter vector L of a CP in expression (14) in accordance with the measurement M data.

When a set-based solution is derived, the following methodology is implemented: a number of polyhedron vertices and faces is limited in a parameter space and dimensionalities of the respective matrices are lowered. The idea of the methodology is to single out the so called inequalities with weak information at each group step. An inequality with weak information is an inequality for which the foremost a priori polyhedron vertex (cut off by a respective half-space) is located close enough to a sectioning hyperplane (in the sense of a chosen criterion). Such inequalities may not be considered at a certain stage, although they are not completely removed, and they will not generate new polyhedron vertices and faces in this case. The set-theoretic estimation of a solution will be rougher but guaranteed, and the problems with "damned dimensionalities" will not emerge. A roughed estimate can be clarified at further steps also with the use of inequalities with a weak information at previous inequality steps.

Table 3 shows the interval-set results for the modeled linear static CP of expression (14) in the 3D parameter space under the chaotic disturbances like random (M [5, 10, 30, 50, 100] measurements), random, aviation noise, Lorenz attractor (100 measurements). The CP identification results are analyzed below for the random-type disturbances.

Initial Guaranteed Interval Estimate (the random-type disturbance, $M = 100$). The number of the one-sided inequalities in system (16) is $N\_ineq = 10100$. The obtained initial guaranteed interval parameter vector estimate is characterized by the parallelepiped center vector $centr\_i$ and by the vector of admissible coordinate deviations ($x\text{mod}\_i$) from a center. The closeness measure of $centr\_i$ to a real parameter value $\tilde{L}$ in expression (17) is the relation $\|\tilde{L} - centr\_i\| / \|\tilde{L}\| = 0.509$, and the measure of deviation of coordinates of points that belong to an initial guaranteed interval estimate is the relation $\|x\text{mod}\_i\| / \|\tilde{L}\| = 4.182$ (where $\|\cdot\|$ is the Euclidean vector norm**)**. Check the informativity of system (16) with respect to an initial interval estimate, and $N\_noninf\_i = 909$ inequalities turn out to be not with information.

Set-Theoretic Estimation (the random-type disturbance $M = 100$). The time, during which the true set-theoretic solution to system, (16) is $t\_solv = 42\ s$. The number of the non-informative inequalities under the set-theoretic estimation is $N\_noninf\_s = 9156$. Therefore, when there are 10100 initial inequalities, it turns out that the number of the information-carrying ones is only $N\_\inf = N\_ineq - (N\_noninf\_i + N\_noninf\_s) = 10100 - (909 + 9156) = \mathbf{35}$ inequalities. This fact can be explained by the successful choice of an initial interval solution estimate and by the expedient permutations of inequality order in a transformed system. That is why the set-based polyhedron-form solution to the system is obtained rapidly enough. Pursuant to the found set-based solution, the interval estimate is yielded that is constructed as the result of the projection of a derived polyhedron onto coordinate axes. The interval estimate is characterized by the parallelepiped center vector $centr\_s$ and by the vector of admissible deviations of coordinates $x\text{mod}\_s$ from the center. The measure of closeness of $centr\_s$ to a real parameter vector value $\tilde{L}$ in expression (17) is the relation $\|\tilde{L} - centr\_s\| / \|\tilde{L}\| = 0.025$, and the measure for the deviation of the coordinates for the points, that belong to the initial guaranteed interval estimate, is the relation $\|x\,\mathrm{mod}\_s\| / \|\tilde{L}\| = 0.118$. The interval-set identifications are compared with the results obtained according to the least-squares method. The degree of closeness of point solution to the identification problem according to the least-squares method, i.e. of L_{lsq}, to the real value of $\tilde{L}$, is characterized by the relation $\|\tilde{L} - L_{lsq}\| / \|\tilde{L}\| = 0.119$.

Table 3. Interval-Set Identification Results

Random	Initial Guaranteed Interval Estimate					
	M	N_ineq	$\|\tilde{L} - centr\_i\| / \|\tilde{L}\|$	$\|x\,\mathrm{mod}\_i\| / \|\tilde{L}\|$	N_noninf_i	$\|\tilde{L} - L_{lsq}\| / \|\tilde{L}\|$
	5	30	0.357	3.087	0	0.219
	10	110	0.304	7.265	1	0.268
	30	930	0.807	5.587	8	0.194
	50	2550	0.347	4.606	237	0.039
	100	10100	0.509	4.182	909	0.119
	Set-Theoretic Estimation					
	M	N_ineq	t_solv (s)	N_noninf_s	$\|\tilde{L} - centr\_s\| / \|\tilde{L}\|$	$\|x\,\mathrm{mod}\_s\| / \|\tilde{L}\|$
	5	30	1.4	23	0.185	0.614
	10	110	2.8	94	0.318	0.301
	30	930	7.8	922	0.128	0.300
	50	2550	17	2281	0.038	0.161
	100	10100	42.5	9156	0.025	0.118
Aviation noise	Initial Guaranteed Interval Estimate					
	M	N_ineq	$\|\tilde{L} - centr\_i\| / \|\tilde{L}\|$	$\|x\,\mathrm{mod}\_i\| / \|\tilde{L}\|$	N_noninf_i	$\|\tilde{L} - L_{lsq}\| / \|\tilde{L}\|$
	100	10100	3.016	10.97	650	0.057
	Set-Theoretic Estimation					
	M	N_ineq	t_solv (s)	N_noninf_s	$\|\tilde{L} - centr\_s\| / \|\tilde{L}\|$	$\|x\,\mathrm{mod}\_s\| / \|\tilde{L}\|$
	100	10100	121,3	9386	0.016	0.831
Lorenz attractor	Initial Guaranteed Interval Estimate					
	M	N_ineq	$\|\tilde{L} - centr\_i\| / \|\tilde{L}\|$	$\|x\,\mathrm{mod}\_i\| / \|\tilde{L}\|$	N_noninf_i	$\|\tilde{L} - L_{lsq}\| / \|\tilde{L}\|$
	100	10100	1.606	4.850	2227	0.090
	Set-Theoretic Estimation					
	M	N_ineq	t_solv (s)	N_noninf_s	$\|\tilde{L} - centr\_s\| / \|\tilde{L}\|$	$\|x\,\mathrm{mod}\_s\| / \|\tilde{L}\|$
	100	10100	64.8	7840	0.050	0.432

Figure 5 depicts the polyhedron-type guaranteed estimate for the parameter set of the CP from expression (14). The random-type disturbance exerts its influence onto it after finding of the intersection of only the first inequality of system (16) with the initial guaranteed interval parameter set estimate. Figure 6 represents the whole polyhedron-form guaranteed CP parameter set for the case with the random disturbance. And Figure 7 shows the whole guaranteed CP parameter set that is in the presence of aviation noise disturbance and figure 8 depict the whole guaranteed CP parameter set under the Lorenz attractor disturbance.

The following conclusions follow from the set-interval identification analysis results. When a number of measurements is large, then, for any disturbance type, an a posteriori interval solution estimate centre approximates a real parameter vector value in a better way than an a priori interval estimate centre and than an estimate obtained according to the least-squares method. When a number of measurements increases, a point solution accuracy increases when a solution is represented by an a posteriori interval estimate, and an accuracy of a solution, obtained according to the least-squares method, decreases. An expedient choice of an initial interval estimate and a change in a system inequality order provide non-informativity of an essential portion of inequalities, and the result is the accelerated process during which a guaranteed CP parameter set is obtained.

Conclusion

The developed Interval-Set Analysis Toolbox in MATLAB is reinforced by the new functions and it is used to solve the problem concerned with modeling of linear static COs under an influence of chaotic disturbances, as well as the problem concerned with interval-set identification of these COs. The paper obtains the guaranteed estimates for the characteristics of some chaotic process classes in the form of the two-side restrictions imposed onto average process values on different averaging intervals that belong to a specified process connexity interval. The derived guaranteed estimates are used to generate the system of the inequalities from which one can obtain a guaranteed CP parameter estimate set in accordance with measurement data. Under stochastic disturbances, the CP identification problem is resolved within the framework of the following realistic uncertainty principle: "inaccurate data – not unique (inaccurate) system". The identification problem solution is reduced to the interval-set linear inequality system one. The Toolbox functions help to implement the efficient algorithms with the use of which an initial guaranteed interval estimate of a parameter set is obtained and a real guaranteed CP parameter set is found under different-class chaotic disturbances. The proposed algorithms, used to estimate chaotic process characteristics and to identify CPs under chaotic disturbances and that are implemented by the Interval-Set Analysis Toolbox, can find their wide practical application areas in which the authors already have the theoretical decisions and the MATLAB-based designs. These areas are: digital adaptive and robust control systems (including real-time systems) under non-stochastic uncertainty conditions; information measurement and testing systems; experiment scheduling and control systems; systems used to analyze and foresee a spacecraft motion trajectory; investment project risk analysis and project efficiency prognostication.

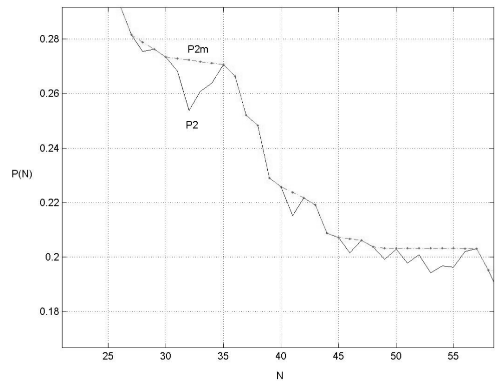

Fig. 1. $p_{2\,m}(N)$ majorant graph plotting process for white noise

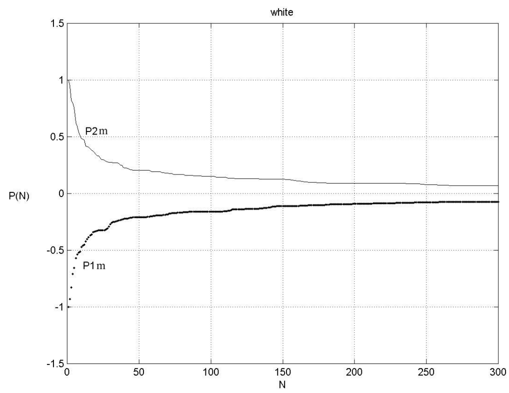

Fig. 2. $p_{1\,m}(N)$ and $p_{2\,m}(N)$ for white noise

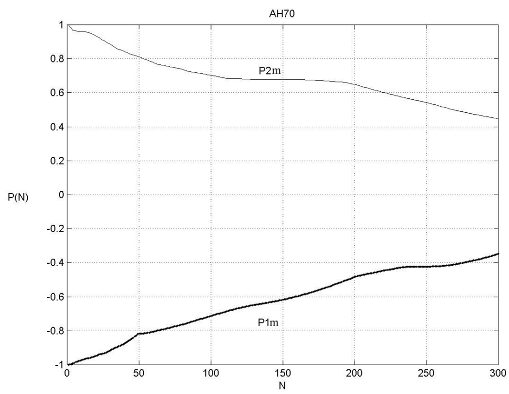

Fig. 3. $p_{1\,m}(N)$ and $p_{2\,m}(N)$ for aviation noise

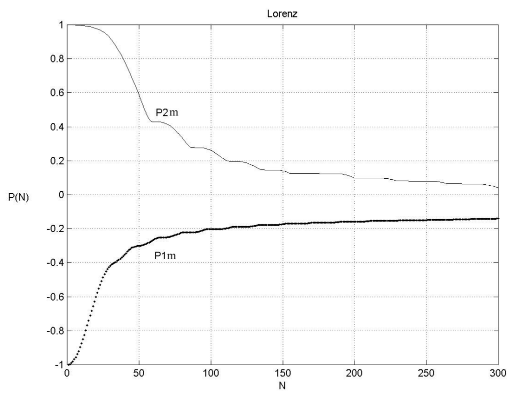

Fig. 4. $p_{1\,m}(N)$ and $p_{2\,m}(N)$ for Lorenz attractor

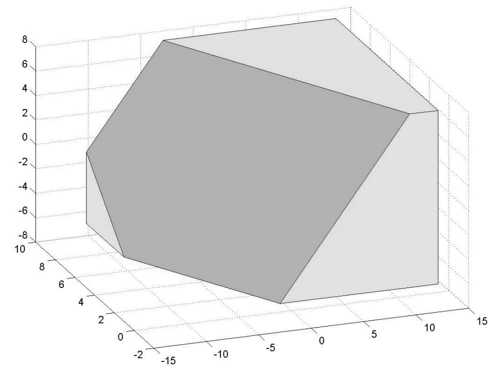

Fig. 5. A guaranteed CP parameter set estimate at the first step under the random type disturbance

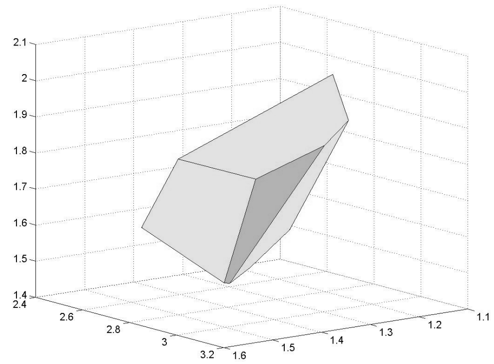

Fig. 6. The whole guaranteed CP parameter set under the random type disturbance

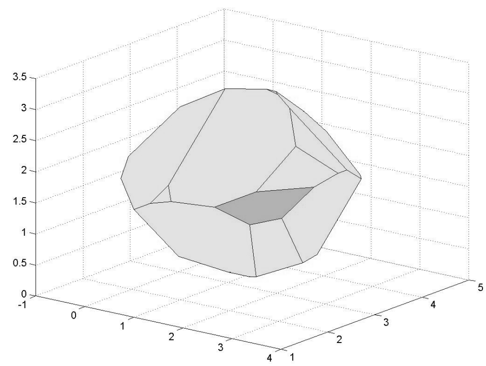

Fig. 7. The whole guaranteed CP parameter set under the aviation noise-type disturbance

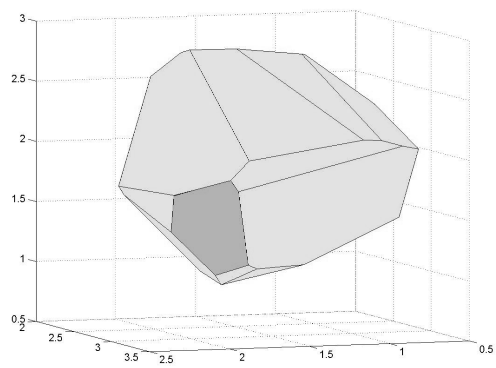

Fig. 8. The whole guaranteed CP parameter set under the Lorenz attractor-type disturbance

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