Numerical Algorithms for Polynomial Matrices *

Martin Hromčík and Michael Šebek

Institute of Information Theory and Automation
182 08 Prague, Czech Republic
{hromcik,msebek}@utia.cas.cz

This report is devoted to new numerical methods for computations with polynomials and polynomial matrices that are encountered when solving the problems of control systems design via the algebraic methods. A distinguishing feature of our approach is the extensive employment of the discrete Fourier transform techniques, namely of the famous Fast Fourier Transform routine and its relation to polynomial interpolation and $Z-$ transform. In view of the availability of powerful computational FFT tools the new algorithms constitute a significant improvement over existing computational methods.

The usability of the proposed methodology has been studied in the context of the following polynomial matrix problems:

- Our algorithms for rank, determinant and adjoint of polynomial matrix have proved their advantages in comparison with existing methods. They appear both more accurate and namely much faster than their predecessors [2, 3, 6]. These algorithms have been programmed and their codes have become a part of the new version 2.0 of Polynomial Toolbox for Matlab [1].

- The case of linear polynomial matrix equations seems to be rather complicated in the view of our approach. A modification of the general scheme is proposed, however, its practical applicability seems problematic [2].

  Nevertheless, an algorithm for factor extraction has been introduced as a special case of polynomial matrix equation which has been shown elegant and more efficient in comparison with Sylvester methods [2].

- The relationship of 2-D DFT to computations with polynomial matrices in two variables has been established and has given rise to a new method for the determinant of 2-D polynomial matrix [4].

  For its importance in robust control problems, this algorithm has also been programmed and the code belongs to Polynomial Toolbox 2.0 [1].

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- All the cases mentioned above are based upon the same basic idea of employing complex points equidistantly distributed on the unit circle and subsequent interpolation through these points.

  Moreover, the FFT is also closely related to the $Z-$transform and spectra of discrete signals. Taking these techniques into account along with the deep results of theory of complex valued functions, an algorithm for discrete-time scalar polynomial spectral factorization has been developed that overtakes existing alternative practical methods [5].

The results of this research find direct practical application in the form of computational macros included in the Polynomial Toolbox for Matlab [1]. This software product appears a most useful tool for teachers, practicing engineers and all other experts dealing with control, signal processing and applied mathematics problems involving polynomials and polynomial matrices.

References

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