TOOLBOX "INTERVAL-SET ANALYSIS" AND ITS APPLICATIONS

Yarema Zyelyk, Mykhailo Lychak
Space Research Institute of National Academy of Sciences and National Space Agency of Ukraine
(40 Academician Glushkov ave. 03680, MSP, KYIV, UKRAINE, tel. + 38 044 2661291,
E-mail: adapt@space.is.kiev.ua)

Now new direction in mathematics named as the interval-set analysis intensively develops. Object in the interval-set analysis is not the point, line or surface, but is general set in some space. The theory of evolution of sets, in which the change of sets in time is described by some difference equations arises. On the basis of the interval-set analysis at problem statement and solving the initial information is given as sets (intervals) and final result of computing algorithm will be some set.

If the interval-set model of uncertainty (unstochastic) is constructed, then mathematical techniques of the interval-set analysis are used for control problem statement and solving in presence of uncertainty. A central moment of such approach is the assumption that for a vector of parameters of the object some a priori estimation as set in space of parameters, to which values of parameters belong, is known. It is similarly assumed, that the values of disturbance vector belong to some known sets as well.

Under control in conditions of uncertainty problems of set parametric identification and set filtration arise. These problems consist of definition of a posteriori set assessments of a parameter vector (state vector) in many-dimensional spaces and of their intersections with a priori estimations. Such estimations are created on data of measurements of input and output values. The measurements are obtained with accuracy to within some noise, for which the a priori estimations are known. In linear case problems of identification and filtration are reduced to definition (on each step) of set solution of the system of the linear algebraic equations with set uncertainty in left and right it parts.

The theory of control under uncertainty with the use set theoretic model is carried out for a long time by the personnel of Space Research Institute of National Academy of Sciences & National Space Agency of Ukraine, which has been separated from V.M. Glushkov Institute of Cybernetics of National Academy of Sciences of Ukraine.

Some previously obtained results have been considered in V.M. Kuntsevich, M.M. Lychak's book, entitled " Guaranteed Estimates, Adaptation and Robustness in Control Systems (Berlin: Springer-Verlag, 1992, 209 p.).

Authors in the above-mentioned direction in 1998-2000 within the framework of scientific project have obtained the fundamental theoretical results. The title of project was "Development and Research Discrete Adaptive Control Algorithms on the Basis of Set Estimates". This Project was carried out at sponsorship of Science and Technology Center in Ukraine (STCU), established by the USA, Ukraine, Canada and EU. These results were presented in many scientific reports, in 12 papers in special scientific international journals and were read at international symposia.

Within the framework of the above Project, algorithm was simulated and examined with the use of the MATLAB and SIMULINK system software means, designed by The MathWorks, Inc. (Natic, USA) specifically for scientific and engineering computations, visualization and simulation. In accordance with the contract, established with the HUMOSOFT s.r.o. (Prague, Czech Republic), the concurrent licensed MATLAB- and SIMULINK-family software products were obtained.

As a further development of researches the authors began to create the new MATLAB Toolbox of "Interval-Set Analysis". The initial version of this Toolbox and his application is a subject of the represented report.

In every MATLAB kernel Toolboxes (in particular, in the ones, aimed at control systems, system identification and so on) an uncertainty, characteristic of an object or a control system, is stochastic. Statistical features of noises and disturbances are assumed to be known (for example probability distribution function very often is normal). Therefore the development of "Interval-Set

Analysis" Toolbox, which will have direct application in the tasks of an estimation, filtration and control in conditions when uncertainty is given by set, is urgent and necessary.

At present, the kernel MATLAB has no functions of set specification such as hyperbands, bars, polyhedra or ellipsoids in multidimensional space. Also the problems, concerned with intersection of polyhedra both with each other and by hyperplanes in N-dimensional space, still remain unresolved within MATLAB, because of absence of methodology and appropriate Toolboxes. However these tasks have fundamental importance for solution of problems of identification, filtration and control based on set models of uncertainty.

Therefore at development of a computer process of simulation of such systems with MATLAB in linear case the important role belongs to initialization of such objects as a hypercube, hyperparallelepiped, convex polyhedron, half-space bounded by a hyperplane. The solution with the tools of MATLAB of the problem of an intersection of a convex polyhedron with a hyperplane in a multydimensional vector space play the main role as well.

With use of functions representing the initial version of the created Toolbox, these problems can be solved easily.

Further are represented a structure and contents of directories of initial "Interval-Set Analysis" version Toolbox.

The INITIAL directory contains functions, with the help of which in N-dimensional spaces are specified a hypercube, hyperparallelepiped, half-space bounded by hyperplane. The initialization of a hyperparallelepiped in space of large dimensionality (more than 10) takes a lot of time and requires significant(sizeable) volumes of the main memory. So the calculation of a matrix of tops of a single cube at M = 14 takes more than 6 min, and at M = 15 - there are more than 20 min on the computer Pentium II 400. Therefore matrices initializing above-stated structures can be once recorded in appropriate data files, and for further use they can be fast read out from files.

The names of data files have a special format indicated in the table DATA.

In the INTERPOLY directory are contained functions, with the help of which the problem of finding of an intersection of a convex polyhedron with a hyperplane in a multidimensional vector space is solved. At implementation of this process the history of a construction of sequence of a posteriori polyhedrons with the help of interceptions by sequence of hyperplanes can be recorded in the appropriate file. Then the process can be reproduced because of data of a saved history.

The DEMOS directory contains programs - scripts to computer simulation of the multistage process of cut-off parts from a remaining polyhedron by a sequence of hyperplanes. At implementation of this process the history of a construction of sequence of a posteriori polyhedrons with the help of interceptions by sequence of hyperplanes can be recorded in the appropriate file. Then the process can be reproduced because of data of a saved history.

Structure of Directories and List of Functions

INTERVAL SET

INITIAL	
vcub	Construction a matrix of vertices of an unit M-dimensional hypercube
permsun	Generation an array of only unique permutations of specified vector elements
vcub_bit	To construct a matrix of vertices of an unit M-dimensional hypercube
gparal	Construction a matrix of faces of such M-dimensional hyperparallelepiped, a vertex matrix for which is adequate to a matrix of vertices of unit M-dimensional hypercube
V_G_CUB_FILE	The script-file of generating of vertexes matrix of M-dimensional unit hypercube with use of a function of unique permutations of (M-1) – dimensional vector elements, formation of a matrix of faces of a hypercube and writing these matrixes in files with entered names
V_G_read_auto	To read matrix of vertexes or faces of a hypercube of specific dimension from the file, which name is determined automatically
V_G_disp_file	The script-file of displaying matrix of vertexes or matrix of faces of M-dimensional unit hypercub, inputting from file
boudparal	Input of constrains on modulus of each coordinate of vector in M-dimensional space, which is needed to specify a hyperparralelepiped
center	To input coordinates of hyperparallelepipeds center
ineqparal	Specifying an a priori uncertain hyperparallelepiped in a M-dimensional space by a linear inequality system
hyperplane	Specifying parameters of hyperplane, generated by random data simulation of current measurement of linear object output

DATA	
v<dimension>.dat	matrix of unit M-dimensional hypercube vertices
g<dimension>.dat	matrix of unit M-dimensional hypercube faces
bound<dimension><name>.dat	vector of constraints at modulo of elements, specifying M-dimensional hyperparallelepiped
center<dimension><name>.dat	vector of center coordinates of M-dimensional hyperparallelepiped
ineq<dimension><name>.dat	matrix **A** and vector **D** in the inequality $\mathbf{A}\,\mathbf{X} \le \mathbf{D}$, specifying a posteriori polyhedron in M-dimensional space
v<dimension><name>.dat	matrix of a posteriori vertices in M-dimensional space
g<dimension><name>.dat	matrix of a posteriori polyhedron faces in M-dimensional space

INTERPOLY	
infoineq	Finding an a priori polyhedron vertex, maximally cut off by a cutting-off hyperplane and its number, and a vertex, minimally cut off by a hyperplane
interpolytop	Finding an intersection of an a priori uncertain polyhedron with a hyperplane, constructed in accordance with measurement data
fifunc	Calculation a value of scalar function used to establish belonging of a selected point in multidimensional space to a halfspace, or for a hyperplane itself
adjacind	Finding numbers of polyhedron vertices, adjacent with respect to a selected vertex
newvertex	Determination coordinates of new vertices of a posteriori uncertain polyhedron, to add them to a matrix of vertices and to add a new row and a new column to a matrix of faces
interceptvert	Generation array of numbers of a priori polyhedron vertices, cut off by a hyperplane
removllcept	Removing index of a next coming vertex, which is cut off, from an array of indices of out-off vertices, to perform respective rejections of columns from vertex and face matrices, to renumbering them
null_lines_G	Removing "0"-lines from matrix of polyhedron faces and deleting appropriate lines from matrix A and appropriate elements of vector D in system inequality $\mathbf{A}\,\mathbf{X} \le \mathbf{D}$, specifying a posteriori polyhedron
apostineq	Constructing system of linear inequalities, specifying a polyhedron of an a posteriori uncertainty in M-dimensional space
write_iter_c_b	Write the number of iteration and parameters of the next intersecting polyhedron hyperplane in the file on each step at simulation of multistage cut-off process of parts from remaining polyhedron
read_iter_c_b	To read number of iteration and parameters of the next intersecting polyhedron hyperplane from the file on each step at simulation of multistage process of cut-off parts from current polyhedron

DEMOS	
D_POLYTOP_FILE	The demonstration program for simulation of multistage cut-off process of parts from remaining polyhedron by sequence of hyperplanes

AUXILIARY	
FILE_CORR	Script-file for review and correction of the file, containing M-dimensional vector
FACTOR	Script-file for calculation of operative memory size, which is needed to run function **perms**