SIMULATION OF CONTROL OF MULTIVARIABLE SYSTEMS TOGETHER WITH REAL TIME CONTROL OF LABORATORY MODEL USING MATLAB

MAREK KUBALČÍK, VLADIMÍR BOBÁL

TU Brno, Faculty of Technology Zlín
Department of Automatic Control
Nám. TGM 275, 762 72 Zlín
Czech Republic

1. INTRODUCTION

Many technological processes require that several variables relating to one system are controlled simultaneously. Each input may influence all system outputs. The design of a controller able to cope with such a system must be quite sophisticated**.**

This paper presents the design and simulation of adaptive control for a two input-two output system together with the real-time control of a laboratory model using this designed method. The synthesis is based on a polynomial approach

2. A DESCRIPTION OF A TWO INPUT – TWO OUTPUT SYSTEM

The internal structure of the the system is shown in Fig. 1

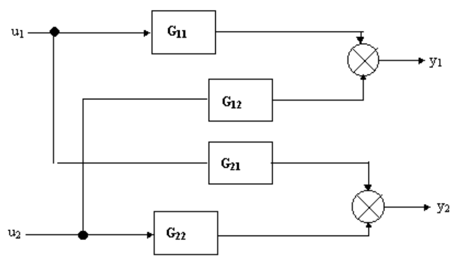

Fig. 1 Double input double output system – "P" structure

Transfer matrix of the system is

$$\boldsymbol{G} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \tag{1}$$

It is possible to assume that the system is described by the matrix fraction

$$\boldsymbol{G}\left(z^{-1}\right) = \boldsymbol{A}^{-1}\left(z^{-1}\right)\boldsymbol{B}\left(z^{-1}\right) = \boldsymbol{B}_1\left(z^{-1}\right)\boldsymbol{A}_1^{-1}\left(z^{-1}\right) \tag{2}$$

Where polynomial matrices $A \in R_{mm}[z^{-1}]$, $B \in R_{mn}[z^{-1}]$ are left indivisible decomposition of the matrix $G(z^{-1})$ and matrices $A_1 \in R_{mm}[z^{-1}]$, $B_1 \in R_{mn}[z^{-1}]$ are right indivisible decomposition of $G(z^{-1})$.

The matrices of the discrete model are

$$\boldsymbol{A}\left(z^{-1}\right) = \begin{bmatrix} 1 + a_1 z^{-1} + a_2 z^{-2} & a_3 z^{-1} + a_4 z^{-2} \\ a_5 z^{-1} + a_6 z^{-2} & 1 + a_7 z^{-1} + a_8 z^{-2} \end{bmatrix} \tag{3}$$

$$\boldsymbol{B}\left(z^{-1}\right) = \begin{bmatrix} b_1 z^{-1} + b_2 z^{-2} & b_3 z^{-1} + b_4 z^{-2} \\ b_5 z^{-1} + b_6 z^{-2} & b_7 z^{-1} + b_8 z^{-2} \end{bmatrix}$$

3. DESIGNING FEEDBACK CONTROL

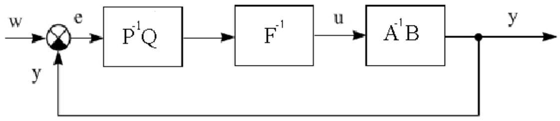

Fig. 2. Block diagram of the closed loop system

In the same way as the controlled system, the transfer matrix of the controller takes the form of matrix fraction

$$G(z^{-1}) = P^{-1}(z^{-1})Q(z^{-1}) = Q_1(z^{-1})P_1^{-1}(z^{-1}) \tag{5}$$

The matrix of an integrator for permanent zero control error is

$$F(z^{-1}) = \begin{bmatrix} 1-z^{-1} & 0 \\ 0 & 1-z^{-1} \end{bmatrix} \tag{6}$$

The control law apparent in the block diagram (operator z^{-1} will be omitted from some operations for the sake of simplification) has the form

$$U = F^{-1}Q_1P_1^{-1}E \tag{7}$$

It is possible to derive the following equation for the system output

$$Y = A^{-1}BF^{-1}P^{-1}QE = A^{-1}BF^{-1}P^{-1}Q(W-Y) \tag{8}$$

which can be modified to give

$$Y = P_1(AFP_1 + BQ_1)^{-1}BQ_1P_1^{-1}W \tag{9}$$

The closed loop system is stable when the following diophantine equation is satisfied

$$AF\,P_1 + BQ_1 = M \tag{10}$$

where $M(z^{-1}) \in R_{mm}[z^{-1}]$ is a stable diagonal polynomial matrix.

$$M(z^{-1}) = \begin{bmatrix} 1+m_1z^{-1}+m_2z^{-2}+m_3z^{-3}+m_4z^{-4} & 0 \\ 0 & 1+m_5z^{-1}+m_6z^{-2}+m_7z^{-3}+m_8z^{-4} \end{bmatrix} \tag{11}$$

The roots of this polynomial matrix are the ruling factor in the behaviour of the closed loop system. They must be inside the unit circle if the system is to be stable.

The degree of the controller matrices polynomials depends on the internal properness of the closed loop. The structure of matrices P_1 and Q_1 was chosen so that the number of unknown controller parameters equals the number of algebraic equations resulting from the solution of the diophantine equations using the uncertain coefficients method.

$$P_1(z^{-1}) = \begin{bmatrix} 1+p_1z^{-1} & p_2z^{-1} \\ p_3z^{-1} & 1+p_4z^{-1} \end{bmatrix} \tag{12}$$

$$Q_1(z^{-1}) = \begin{bmatrix} q_1+q_2z^{-1}+q_3z^{-2} & q_4+q_5z^{-1}+q_6z^{-2} \\ q_7+q_8z^{-1}+q_9z^{-2} & q_{10}+q_{11}z^{-1}+q_{12}z^{-2} \end{bmatrix}$$

The solution to the diophantine equation results in a set of sixteen algebraic equations with unknown controller parameters. The controller parameters are given by solving these equations.

The algorithms designed here were incorporated into an adaptive control system with recursive identification. The recursive least squares method proved effective for self-tuning controllers and was used as the basis for our algorithm.

4. SIMULATION

Matlab + Simulink for Windows (The MathWork, Inc.) were used to create a program and diagrams to simulate and verify the algorithms. There are examples of simulation diagrams in Fig. 3 and Fig 4.

Verification by simulation was carried out on a range of systems with varying dynamics. The control of the model below is given here as our example.

$$A(z^{-1}) = \begin{bmatrix} 1+0{,}3z^{-1}+0{,}1z^{-2} & 0{,}1z^{-1}+0{,}2z^{-2} \\ 0{,}1z^{-1}+0{,}3z^{-2} & 1+0{,}3z^{-1}+0{,}1z^{-2} \end{bmatrix} \tag{13}$$

$$B(z^{-1}) = \begin{bmatrix} 0{,}1z^{-1}+0{,}4z^{-2} & 0{,}9z^{-1}+0{,}4z^{-2} \\ 0{,}6z^{-1}+0{,}2z^{-2} & 0{,}3z^{-1}+0{,}4z^{-2} \end{bmatrix}$$

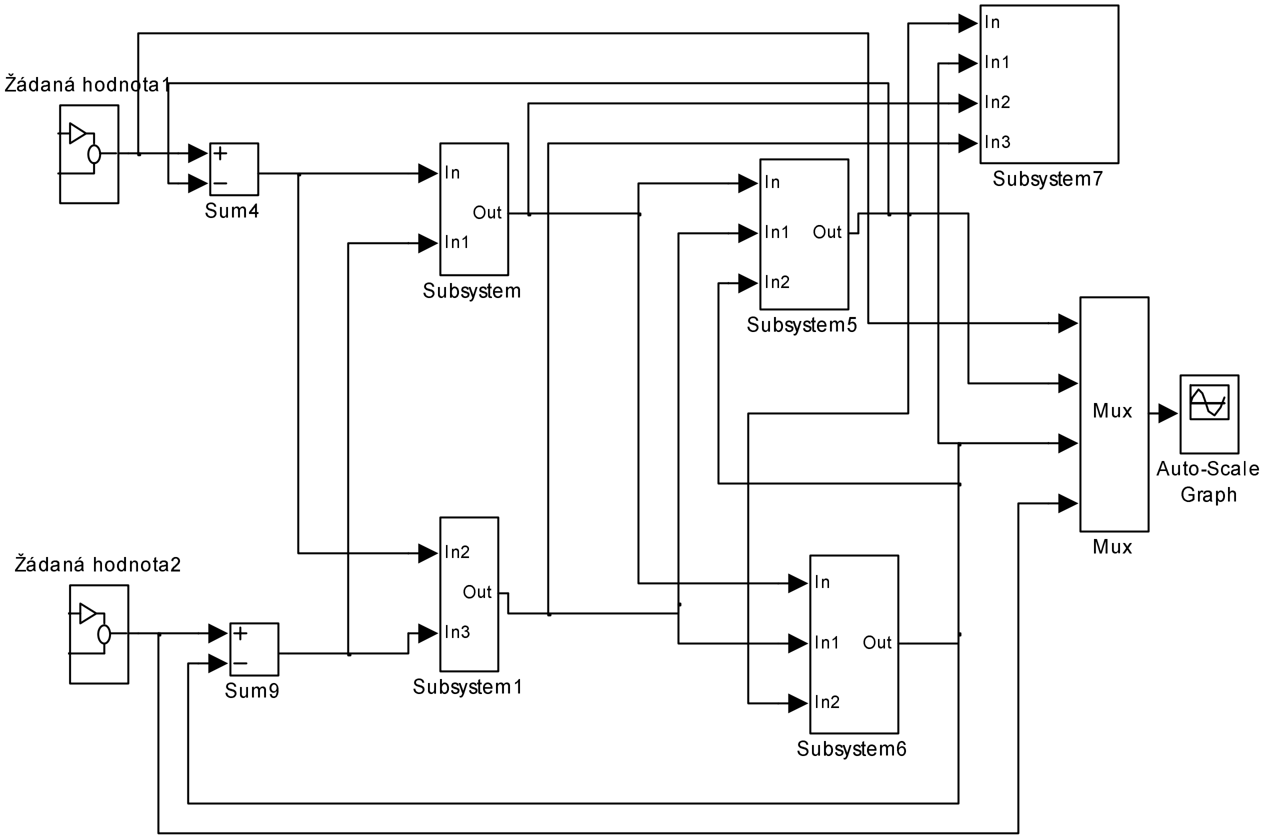

Fig 3 Simulation diagram for adaptive control of the two input – two output system

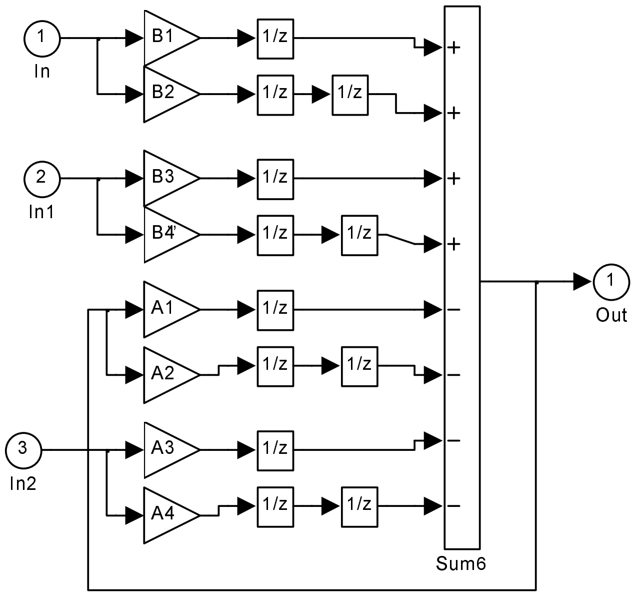

Fig 4 Scheme of the linear discrete systém

The right side control matrix was denoted as follows

$$\boldsymbol{M}_1\left(z^{-1}\right)=\begin{bmatrix}\left(1-0{,}1z^{-1}\right)^4 & 0\\ 0 & \left(1-0{,}1z^{-1}\right)^4\end{bmatrix} \tag{14}$$

Fig. 5 shows the system's step response given by the Polynomial toolbox

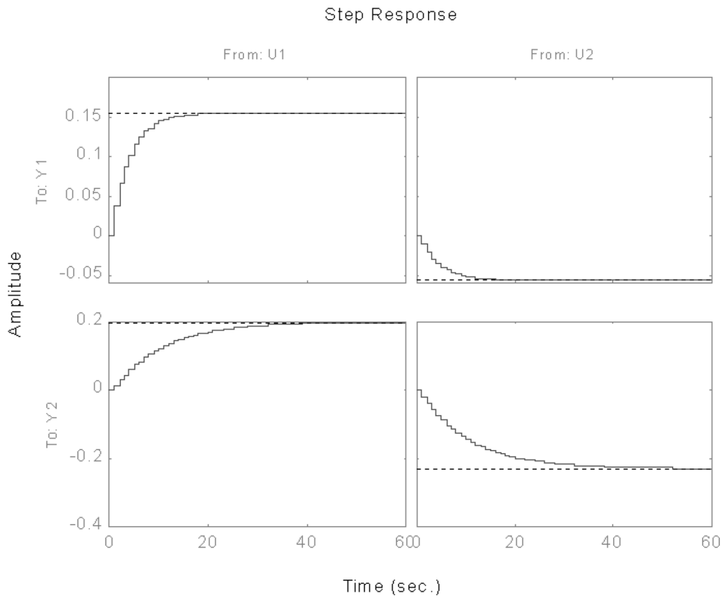

Fig. 5. The step response of the system

The results of simulation are shown in Figs 6 – 7 .

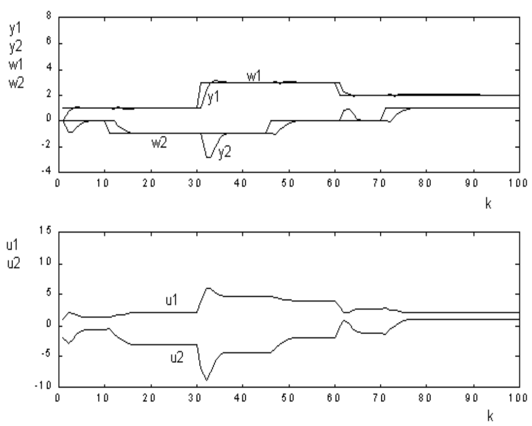

Fig. 6. Deterministic control

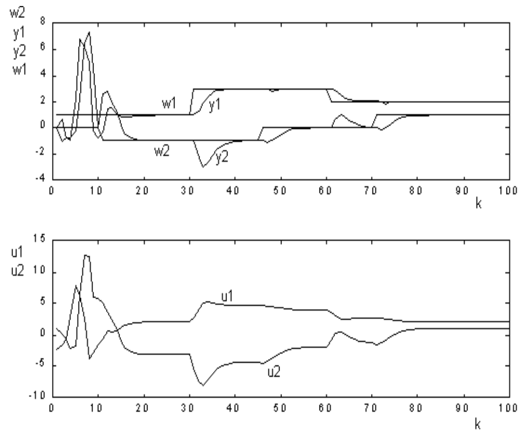

Fig. 7. Adaptive control

5. VERIFICATION – CONTROLLING A LABORATORY MODEL

Our department has experimental laboratory model CE 108 - coupled drives apparatus. This apparatus, based on experience with authentic industrial control applications, was developed in cooperation with the University of Manchester and made by a British company, TecQuipment Ltd. It allows us to investigate the ever-present difficulty of controlling the tension and speed of material in a continuous process. The process may require the material speed and tension to be controlled to within defined limits. Examples of this occur in the paper-making industry, strip metal and wire manufacture and, indeed, any process where the product is manufactured in a continuous strip.

The industrial type material strip is replaced by a continuous flexible belt. The principle scheme of the model is shown in the Fig. (8). It consists of three pulleys, mounted on a vertical panel so that they form a triangle resting on its base. The two base pulleys are directly mounted on the shafts of two nominally identical servo motors and the apparatus is controlled by manipulating the drive torques to these servo motors. The third pulley, the jockey, is free to rotate and is mounted on a pivoted arm. The jockey pulley assembly, which simulates a material work station, is equipped with a special sensor and tension measuring equipment. It is the jockey pulley speed and tension which form the principle system outputs. The belt tension is measured indirectly by monitoring the angular deflection of the pivoted tension arm to which the jockey pulley is attached.

The manipulated variables are the inputs to the servo motors and the controlled variables are the tension and speed at the work station. There are interactions between the control loops.

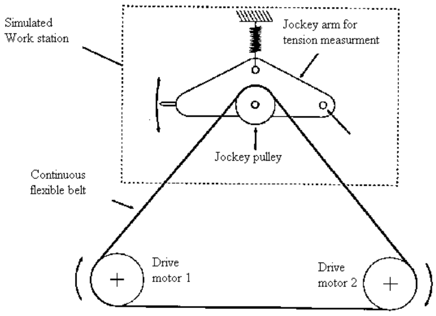

Fig. 8. Principal scheme of CE 108

The task was to apply the method we designed for the adaptive control of a model representing a non-linear system with variable parameters which is, therefore, impossible to control deterministically. Adaptive control using recursive identification was performed. The time responses of the control are shown in Fig. 9 and Fig. 10. The controlled variable y_1 is the speed and the controlled variable y_2 is the tension.
Connection between the model and a computer was realized using the technological adapter Advantech PCL 812. Programs for control of the real model were created in Matlab in the form of M – files with help of the Real Time Toolbox.

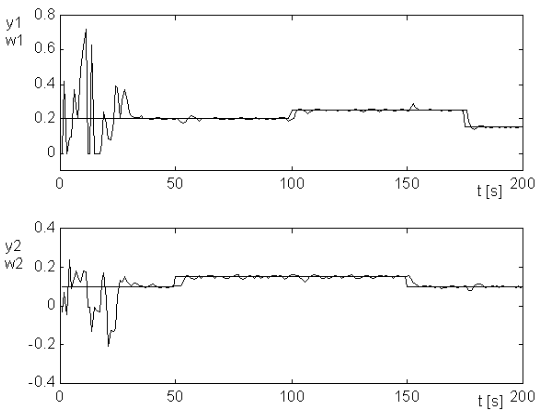

Fig. 9. The adaptive control of a real model

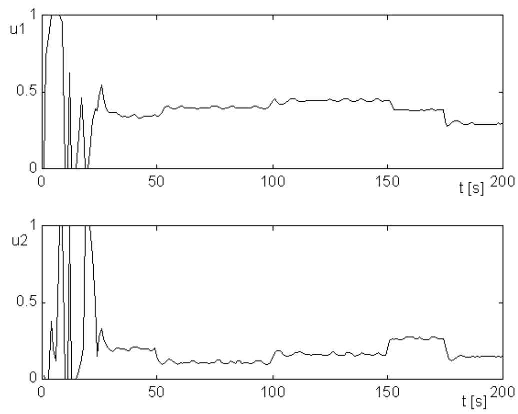

Fig. 10. Controller output

6. CONCLUSIONS

The adaptive control of a two-variable system based on polynomial theory was designed. The design was simulated and used to control a laboratory model. The simulation results proved that this methods is suitable for the control of linear systems. The control tests on the laboratory model gave satisfactory results despite the fact that the non-linear dynamics were described by a linear model. The Matlab and Simulink proved as useful tools both for the simulation and for the laboratory tests.

ACKNOWLEDGMENTS

The work has been supported by the Grant Agency of the Czech Republic under grants No. 102/99/1292 and 102/01/P013 and by the Ministry of Education of the Czech Republic under grant MSM 281100001. This support is very gratefully acknowledged.

REFERENCES

1. Bittanti, S., Bolzern, P., Campi, M. (1990). Convergence and Exponential Convergence of Identification Algorithms with Directional Forgetting Factor. *Automatica*, 26, 5, pp. 929 – 932.
2. Luyben W. L. (1986). Simple Method for Tuning SISO Controllers in Multivariable Systems. *Ing. Eng. Chem. Process Des. Dev*., 25, pp. 654 – 660.
3. Skogestad, S., Postlethwaite, J. (1996) *Multivariable Fedback Control - Analysis and Design.* New York, J. Willey.
4. Real Time Toolbox for Windows. (1994). User`s Manual. Humusoft.
5. Cavalo, A. et al. (1996). *Using Matlab, Simulink and Control System Toolbox.* Prentice Hall.