Numerical Study of Plasma Fluctuations and Their Influence upon Probe Characteristics

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Abstract

Rf or pulsed discharges have attracted a growing interest as a tool for plasma etching and deposition processes. Measurement of current-to-voltage characteristic using a small metallic electrode (Langmuir probe) immersed into the low temperature plasma enables to determine the most important parameters of the plasma like plasma potential, density and temperature of electrons or electron energy distribution function. Fluctuations of plasma parameters lead to distortion of plasma characteristic. The question how these fluctuations influence errors of computed plasma parameters has been so far solved for special forms of fluctuations, leading to explicit mathematical formulas. To overcome these limitations we developed a code, written in Matlab, treating the problem on purely numerical basis. In contrast to previously published papers numeric access enables to investigate the influence of fluctuations of arbitrary shapes, amplitudes and correlations. Some results for stochastic (Gaussian) and deterministic (e.g. rectangular) fluctuations are presented.

1. Theory

A somewhat idealized model of Langmuir probe leads to the Maxwellian electron density current [1]

$$j_0(V,N,T) = N \cdot \sqrt{\frac{eT}{2\pi m}} \cdot \exp\left(\frac{V}{T}\right), \qquad V < 0,$$

$$j_0(V,N,T) = N \cdot \sqrt{\frac{eT}{2\pi m}} \cdot \left(1 + \frac{V}{T}\right)^{1/2}, \qquad V > 0. \tag{1}$$

Here j_0 is the non-disturbed electron density flow, V is the probe potential related to the plasma potential V_s, N is the electron number density and T is the electron temperature measured in volts. From the ideal probe characteristic (1) and mainly from its retarding region $V < 0$ the electron parameters N, T and the plasma potential V_s can be determined in several ways.

The plasma potential V_s is usually established from the second derivative of the probe characteristic. At this point the first derivative as well as the second one are discontinuous. The characteristic course of the second derivative in the vicinity of the plasma potential enables to locate this potential, e.g. as a point, where $j''(V_s) = 0$.

The electron temperature is most readily determined from the slope of the characteristic in the semilogarithmic plot,

$$T_{\log} \equiv \left(\frac{\mathrm{d} \log j}{\mathrm{d} V} \right)^{-1}, \quad V < 0. \tag{2}$$

Knowing the plasma potential, the parameters N_{Fit} and T_{Fit} can be obtained by fitting the electron current to the functional form (1).

The transition point between unsaturated and saturated region simply connects the electron number density with the electron temperature and plasma potential:

$$N_{\mathrm{j0}} \equiv \sqrt{\frac{2\pi m}{k T_{\mathrm{Fit}}}} \cdot j_0(0). \tag{3}$$

Hitherto, the fluctuations of parameters $x \equiv (V, N, T)$ have not been taken into account. The real measured probe current j is given as the average of instantaneous values j_0(x), i.e. $j(<x>) = <j_0(x)>$. The brackets denote an ensemble average for stochastic (e.g. Gaussian) fluctuations and a time average for deterministic (e.g. harmonic) oscillations. Due to the non-linearity of the probe current the dependence of j on $<x>$ is generally very complicated and can be explicitly expressed in only a few special cases, e.g. for small fluctuations enabling Taylor's expansion to the second order [2,3], for Gaussian fluctuations [4] or harmonic oscillations [5] of the probe potential.

2. Numeric Realization and Results

To process the probe characteristics we have developed three programs written in Matlab. The first program models the real probe characteristic as an average of the function (1) over the fluctuations of plasma parameters and probe voltage. The input parameters of this program are mean values of electron number density, electron temperature and plasma space potential. Further types of fluctuations and their standard deviations and correlations are specified.

The second program determines plasma parameters from the probe characteristic under assumption that this characteristic is Maxwellian. The used methods of plasma analysis were described in previous section. To eliminate the random noise from experimentally obtained data and to evaluate the first two derivatives of the probe characteristic the code applies the least-squares approximation by splines from the Splines Toolbox (functions `spap2` and `fnder`). The smoothed characteristic and its derivatives are evaluated as the average over several splines with different knot sequences. This technique significantly improves numeric reliability. The nonlinear least squares fitting determining the electron number density and electron temperature is performed by the function lsqcurvefit from the Optimization Toolbox.

The last program computes the plasma parameters from the characteristic distorted by rectangular probe potential fluctuations. Contrary to the previous program these fluctuations are taken into account from the beginning by special procedure, which will be published later.

The sensitivity of the probe measurements to the fluctuations is studied on modeled examples. First, the probe characteristic affected by various plasma and probe fluctuations is numerically simulated. Second, this characteristic is treated as noiseless and the plasma potential, electron density and temperature are computed. The differences between computed and input plasma parameters give quantitative errors brought in by fluctuations.

In following two examples we consider the distortions of the probe current (1) with parameters $N = 1 \cdot 10^{16}$ m^{-3}, $T = 1$ eV.

As a first example let us consider Gaussian probe potential fluctuations described by the probability density function (pdf)

$$\rho(v) = \frac{1}{\sqrt{2\pi} \cdot \sigma_V} \cdot \exp\left(-\frac{v^2}{2\sigma_V^2} \right), \tag{4}$$

where v is a voltage noise and σ_V is its standard deviation. For the stochastic processes the time average of the probe current is replaced by the ensemble average

$$j(V) = \int_{-\infty}^{+\infty} j_0(V + v) \cdot \rho(v)\,\mathrm{d}v . \tag{5}$$

Table 1 shows computed density, temperature and plasma potential for some values of σ_V if the distortion caused by fluctuations is ignored and the averaged probe current (5) is treated as noiseless.

$\frac{\sigma_V}{T}$	N_{j0} [10^{16} m^{-3}]	N_{Fit} [10^{16} m^{-3}]	T_{Fit} [eV]	V_s [V]
0.5	0.8	0.9	1.0	-0.2
1.0	0.8	0.9	1.3	-0.1
2.0	0.8	0.8	2.4	0.7
4.0	0.8	0.8	5.4	2.6
10	0.7	0.7	-	-

Tab.1 Distortion of the probe characteristic due to Gaussian fluctuation of the probe potential.

$\frac{A}{T}$	N_{j0} [10^{16} m^{-3}]	N_{Fit} [10^{16} m^{-3}]	T_{Fit} [eV]	V_s [V]
0.5	0.8	0.9	1.0	-0.2
1.0	0.7	0.7	1.4	-0.2
2.0	0.6	0.6	2.6	-0.2
5.0	0.4	0.4	7.5	-0.2
10	0.4	0.4	16	-0.2

Tab.2 Distortion of the probe characteristic due to rectangular oscillations of the probe potential.

As follows from these results, the calculation of the electron density is only slightly dependent on the fluctuation magnitude and its estimation is undervalued of about 20 %. Determination of the plasma potential from the 2nd derivative of the probe characteristic is possible up to $\sigma_V \approx 5T$, above this boundary the second derivative oscillates and its zero point is not defined uniquely (Fig.1). Gaussian fluctuations most strongly influence the temperature evaluation.

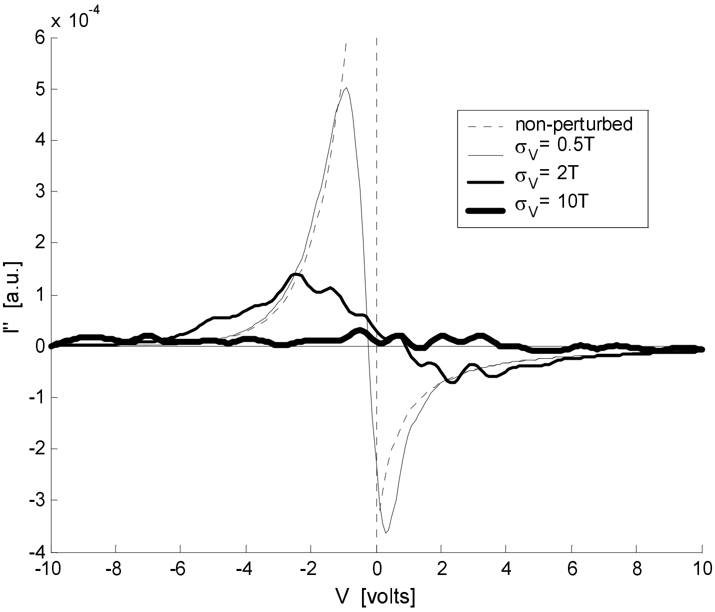

Fig.1 Second derivatives of the probe characteristic distorted by Gaussian fluctuation of the probe potential.

The mean value for deterministic fluctuations $v(t)$ is

$$j(V) = \frac{1}{\tau} \cdot \int_0^{\tau} j_0[V + v(t)]\,\mathrm{d}t\,, \tag{6}$$

where τ is a period of oscillations or a large time interval. As the simplest representative of such signals we chose rectangular oscillations of the amplitude A,

$$\begin{aligned} v(t) &= -A, \quad t \in (0, \tau/2), \\ v(t) &= +A, \quad t \in (\tau/2, \tau), \end{aligned} \tag{7}$$

for which the distorted probe characteristic can be easily expressed explicitly:

$$j(V) = \frac{j_0(V - A) + j_0(V + A)}{2}\,. \tag{8}$$

Results of numeric computations are listed in Tab. 2. Similarly to the previous example, determination of the electron number density from distorted characteristic is in comparison with the electron temperature relatively stable. The independence of the computed plasma potential from the magnitude of oscillations requires some explanation. According to the decomposition in Eq. 8 the second derivative $j''(V)$ contains two characteristic peaks at points $V_1 = V_s - A, \quad V_2 = V_s + A$

(Fig.2; similar course of the second derivative have some other odd oscillations, e.g. harmonic or triangular). Hence the plasma potential and the amplitude of oscillations can be easily determined with high accuracy as

$$V_s = \frac{V_1 + V_2}{2}, \qquad A = \frac{V_2 - V_1}{2}. \tag{9}$$

The systematic error 0.2 V in determination of V_s is due to numeric smoothing of discontinuities at the points V_1, V_2.

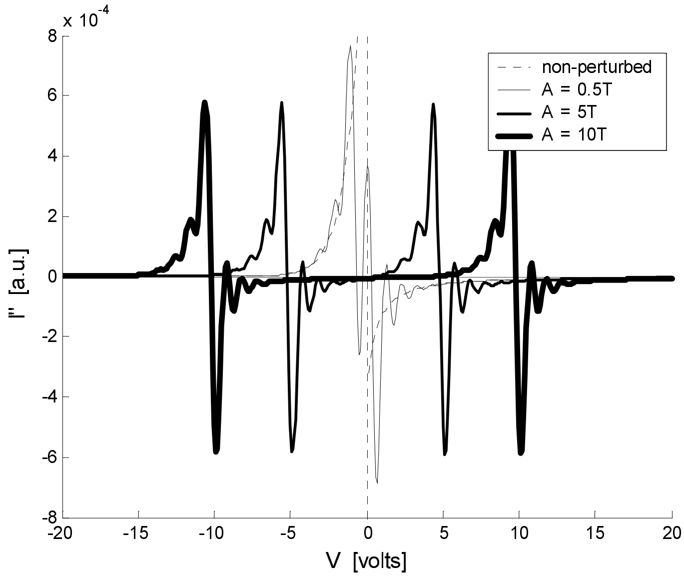

Fig.2 Second derivatives of the probe characteristic distorted by rectangular oscillations of the probe potential.

Figure 3 shows experimental characteristic measured with probe voltage superimposed by rectangular oscillations. After strong smoothing and eliminating the ion current the first and second derivatives were calculated (Fig. 4) and from their maxima or zero points the plasma potential and amplitude of oscillations were established (Eqs. 9): $V_s = -10$ V, $A = 5.5$ V. After this procedure the electron number density and electron temperature could be determined from the probe characteristic: $N = 4.0$, $T = 1.4$. Units are the same as in Tab. 1. The numerically modelled characteristic for computed plasma parameters and amplitude of oscillations is in very good agreement with the experimental one. As the corresponding noiseless characteristic does not match the measured data at all (Fig. 3), it is quite impossible to neglect fluctuations in this example.

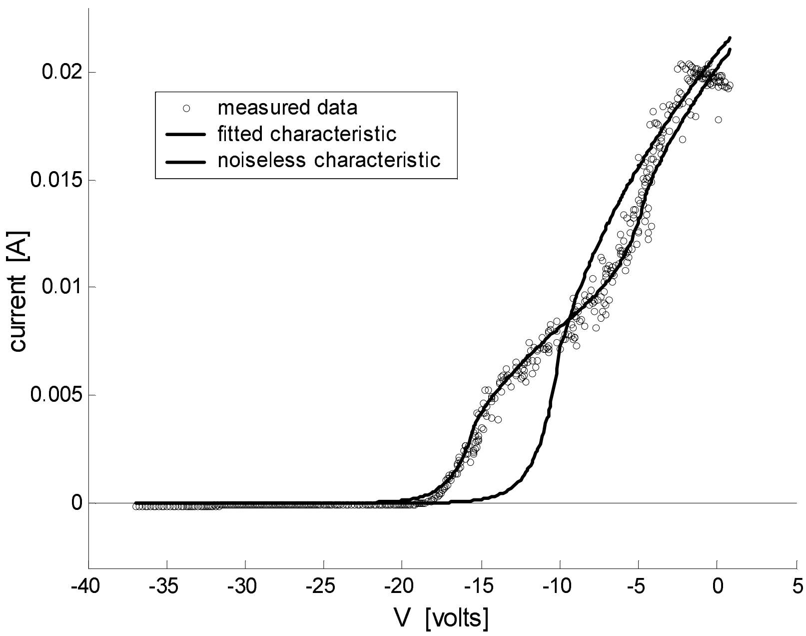

Fig.3 Measured and computed probe characteristics with the probe potential modified by rectangular oscillations.

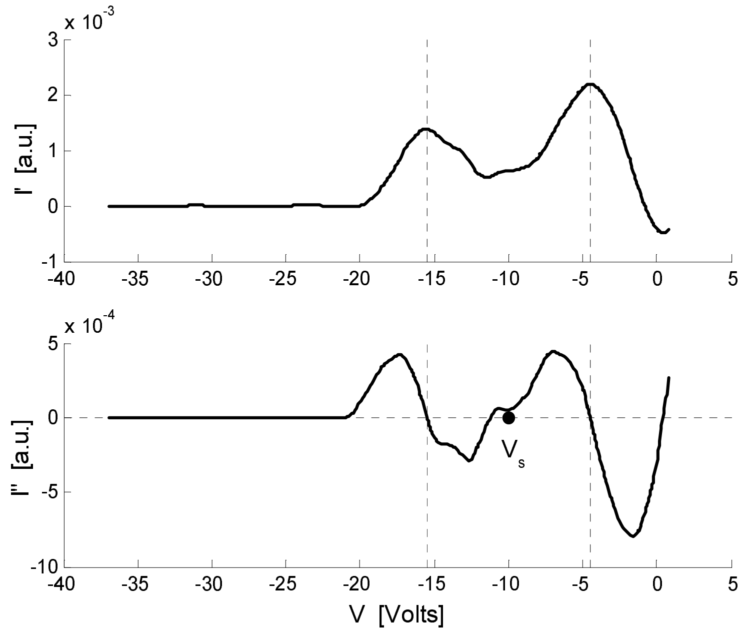

Fig.4 First and second derivatives of measured probe characteristic distorted by rectangular oscillations of the probe potential.

The last test of the probe distortion refers to realistic simultaneous fluctuations of the electron density and temperature [6], as they were theoretically predicted for argon plasma driven by 500 W power modulated by ideal rectangular waves. The results obtained from distorted probe characteristic (in units from Tab.1: $N_{j0} = 2.5$, $N_{\rm Fit} = 2.6$, $T_{\rm Fit} = 3.2$) are in good agreement with real mean values ($< N >= 2.75$, $< T >= 3.24$). In this case the presence of fluctuations can be fully ignored.

3. Conclusion

The authors wrote several Matlab m – files modelling fluctuations in low temperature plasma and determining plasma parameters from the noiseless probe characteristics. The computer modelling of fluctuations overcomes limitations of purely theoretical access because it enables simple testing of the influence of fluctuations on the probe characteristic regardless of their types and magnitudes. Various mathematical functions and graphical instruments offered by Matlab environment were employed to solve these problems efficiently and on a high programming level.

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