Equation-Based Modeling

Agenda

- Equation-based modeling
  - What it is
  - When to use it
- Demo: modeling the efficiency of a mosquito trap
- Interfaces for setting up your own equations
- Q&A

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COMSOL MULTIPHYSICS®

The platform product for simulating real-world designs, devices, and processes. One user interface for all engineering applications.

— MODEL BUILDER: Combine physics phenomena in one model

— APPLICATION BUILDER: Build simulation apps from models

— MODEL MANAGER: Collaborate and organize models and apps

COMSOL Compiler™

Compile simulation apps into executable files. Run them freely on any computer.

COMSOL Server™

Host and administrate your simulation apps. Run them through a web interface.

ADD-ON PRODUCTS

ELECTROMAGNETICS

- AC/DC Module
- RF Module
- Wave Optics Module
- Ray Optics Module
- Plasma Module
- Semiconductor Module

FLUID & HEAT

- CFD Module
  - Mixer Module
- Polymer Flow Module
- Microfluidics Module
- Porous Media Flow Module
- Subsurface Flow Module
- Pipe Flow Module
- Molecular Flow Module
- Metal Processing Module
- Heat Transfer Module

STRUCTURAL & ACOUSTICS

- Structural Mechanics Module
  - Nonlinear Structural Materials Module
  - Composite Materials Module
  - Geomechanics Module
  - Fatigue Module
  - Rotordynamics Module
- Multibody Dynamics Module
- MEMS Module
- Acoustics Module

CHEMICAL

- Chemical Reaction Engineering Module
- Battery Design Module
- Fuel Cell & Electrolyzer Module
- Electrodeposition Module
- Corrosion Module
- Electrochemistry Module

MULTIPURPOSE

- Optimization Module
- Uncertainty Quantification Module
- Material Library
- Particle Tracing Module
- Liquid & Gas Properties Module

INTERFACING

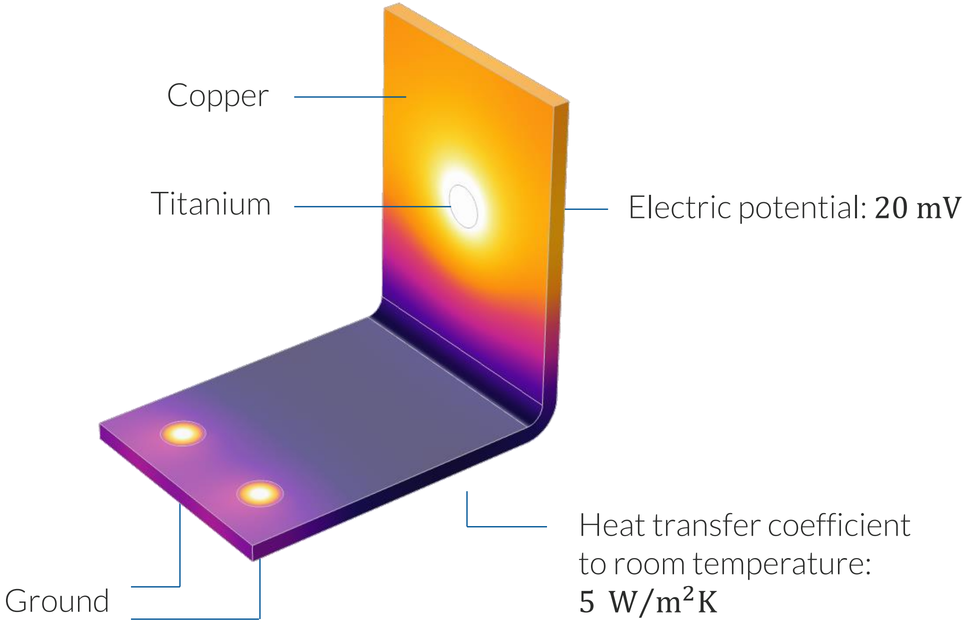
- LiveLink™ *for* MATLAB®
- LiveLink™ *for* Simulink®
- LiveLink™ *for* Excel®
- CAD Import Module
- Design Module
- ECAD Import Module
- LiveLink™ *for* SOLIDWORKS®
- LiveLink™ *for* Inventor®
- LiveLink™ *for* AutoCAD®
- LiveLink™ *for* Revit®
- LiveLink™ *for* PTC® Creo® Parametric™
- LiveLink™ *for* PTC® Pro/ENGINEER®
- LiveLink™ *for* Solid Edge®
- File Import *for* CATIA® V5

EXAMPLE

Modeling with Physics Interfaces

Electrical heating in a busbar

- Electric Currents (ec)
- Heat Transfer in Solids (ht)
- Multiphysics
  - Electromagnetic Heating (emh1)



EXAMPLE

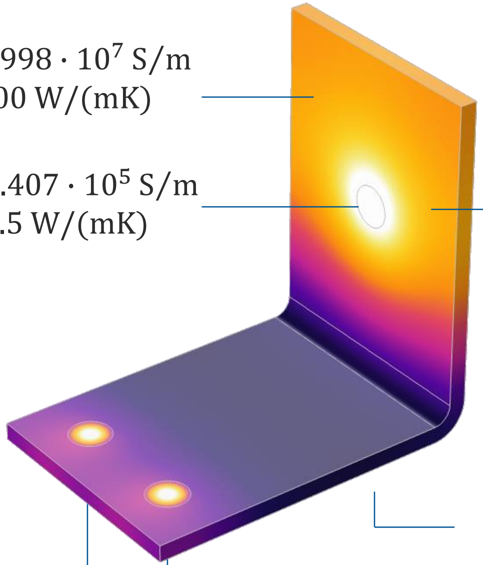
Modeling with Mathematics Interfaces

Electrical heating in a busbar

∇^2 Poisson's Equation (poeq)
∇^2 Poisson's Equation (poeq2)

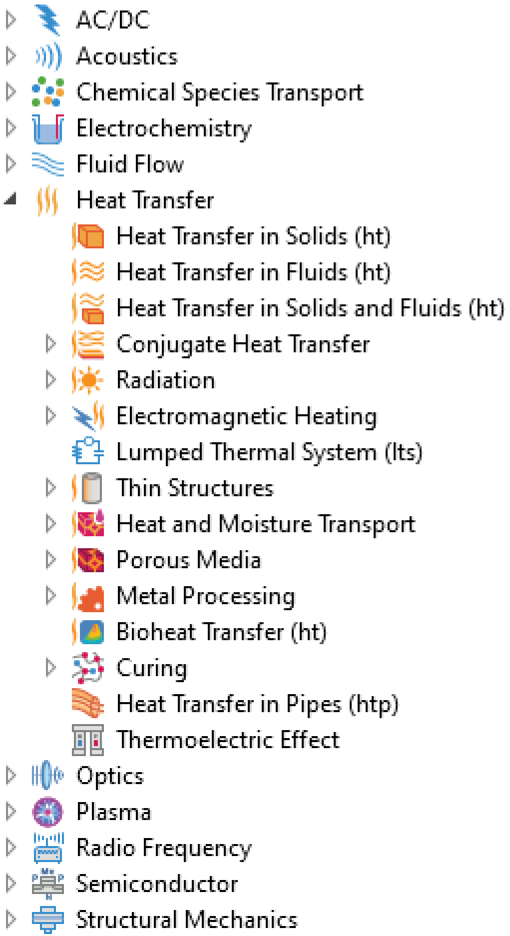
$$\nabla \cdot (-\sigma \nabla V) = 0$$

$$\nabla \cdot (-k \nabla T) = \sigma |\nabla V|^2$$



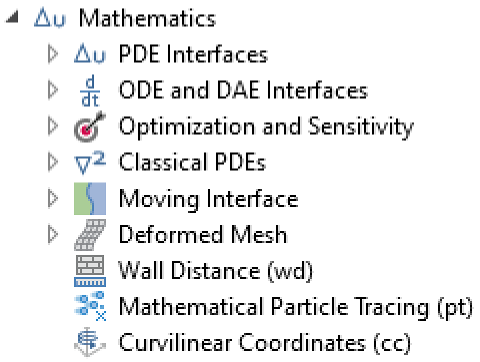
Why Use the Physics Interfaces?

- Pick and choose from >100 physics interfaces covering most scenarios
  - No need to enter the equation
  - Boundary conditions, solver settings, stabilization, variables, plots, etc., for free
- Even if your physics is different, the equation could be the same
  - May be more convenient to adopt or adapt an existing interface than to start from scratch



Why Use the Mathematics Interfaces?

- Extend the functionality of the physics interfaces
  - Constraints
  - Control systems
  - Damage integrals
  - Optimization (with the Optimization Module)
  - Etc.
- Set up your own custom equations
  - Might not resemble existing physics interfaces or even relate to physics



DEMO

Mosquito Trap

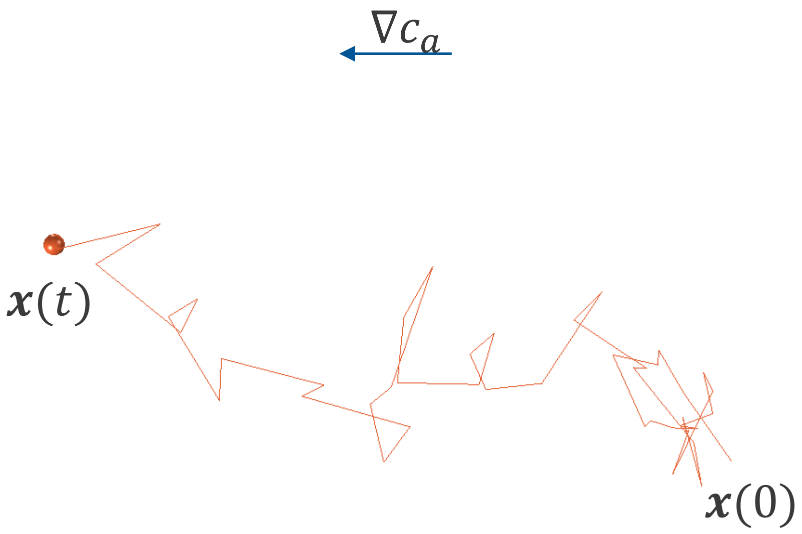
- Absent wind or anything that attracts a mosquito – let us approximate its motion as a 3D random walk

$$\boldsymbol{x_0}(t) = \sum_{n=1}^{t/\Delta t} \boldsymbol{v}_n \Delta t$$

with $\boldsymbol{v}_n$ normally distributed around 0

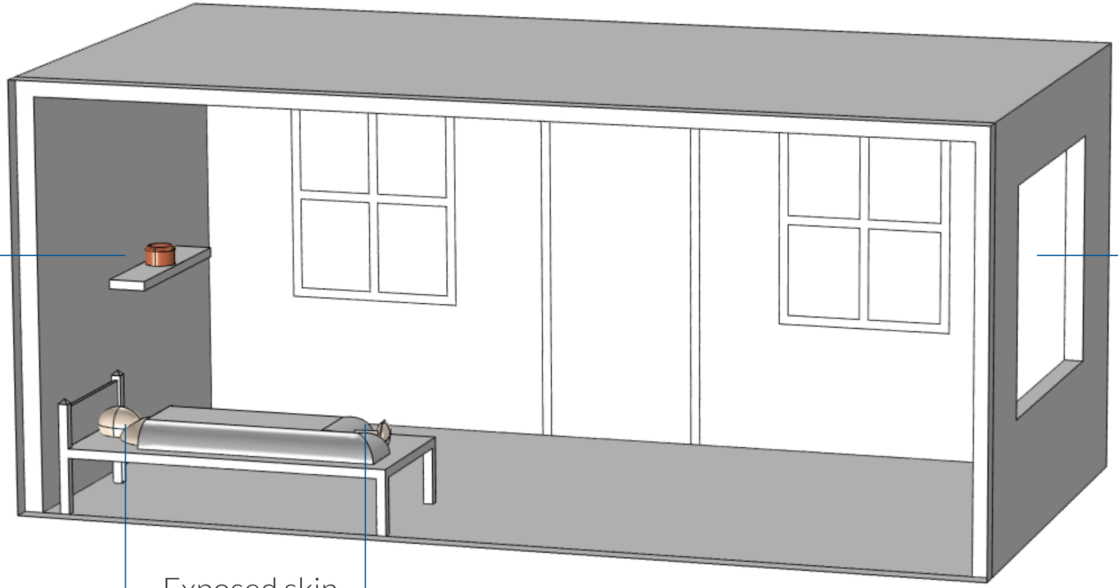
- Introduce an attractor concentration, c_a, and add a tendency to follow its gradient

$$\boldsymbol{x}(t) = \boldsymbol{x}_0(t) + k\int_0^t \nabla c_a \mathrm{d}t$$



DEMO

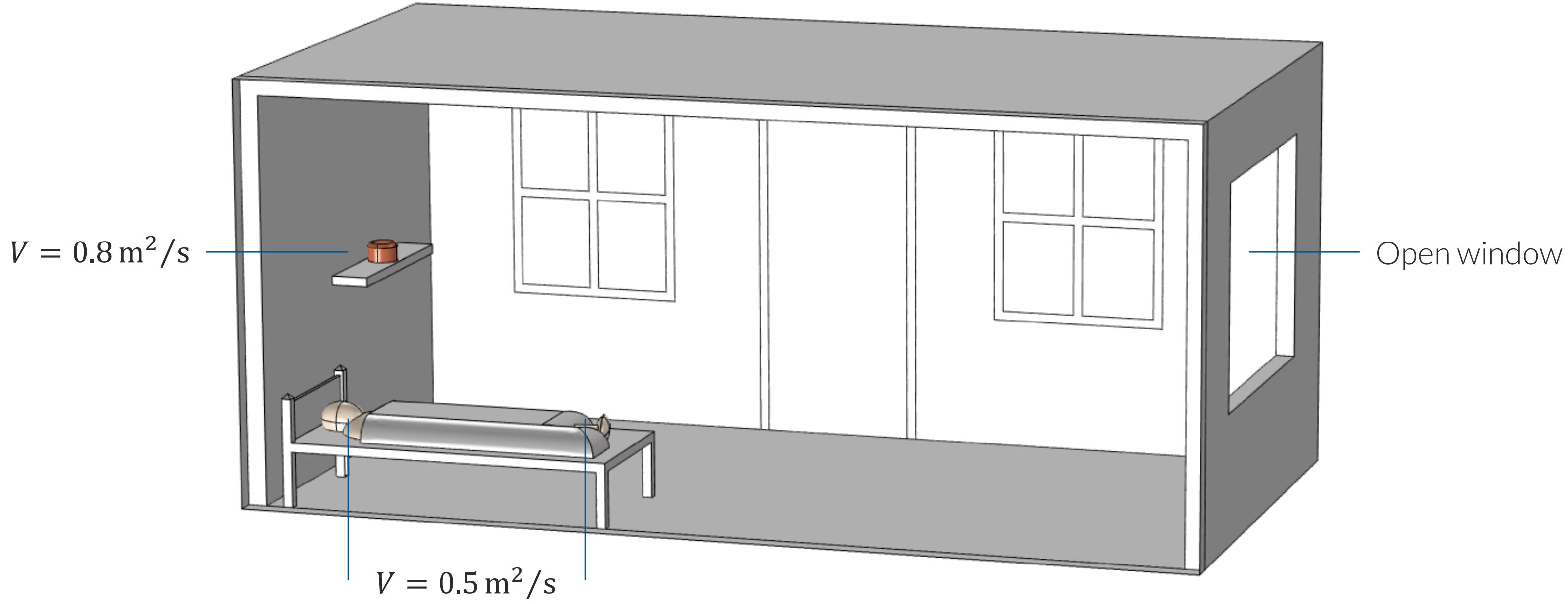
Mosquito Trap



DEMO

Mosquito Trap

Solve $\Delta V = 0$ for velocity potential $V = kc_a$



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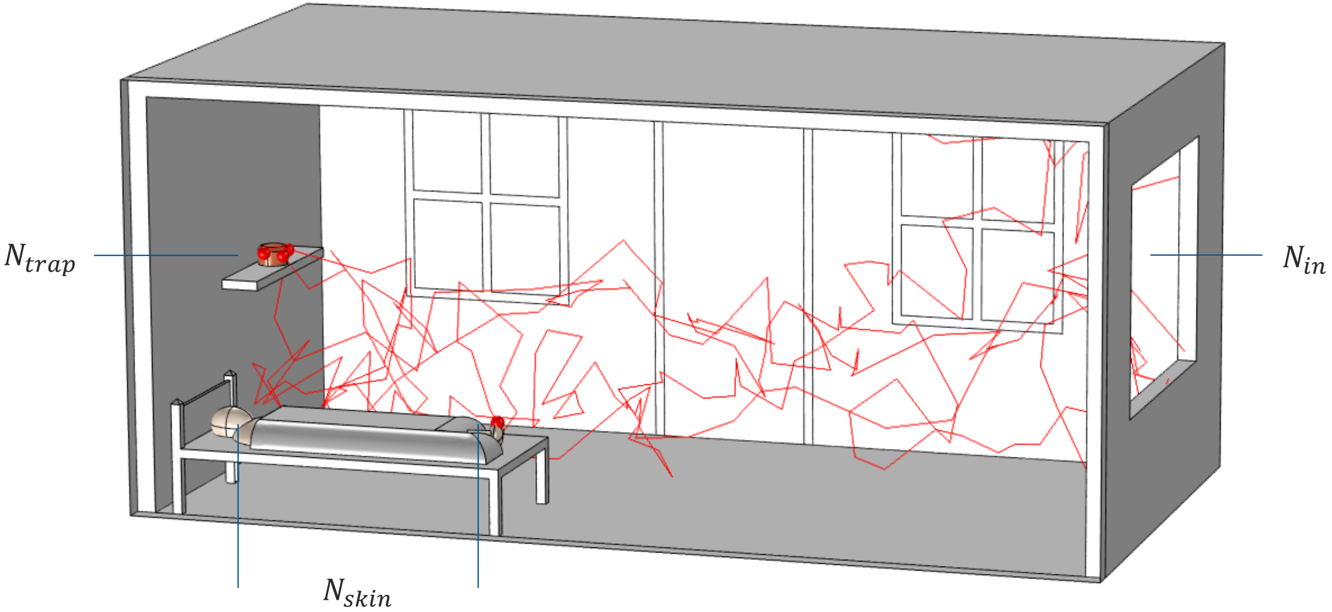
DEMO

Mosquito Trap

With particle tracing, count the number of mosquitoes reaching each destination



DEMO

Mosquito Trap

- Solve an equivalent convection–diffusion equation for the mosquito concentration, c

$$\frac{\partial c}{\partial t} + \nabla \cdot (-D\nabla c) + \nabla V \cdot \nabla c = 0$$

- The diffusion coefficient can be expressed as

$$D = \frac{\sigma^2}{6t}$$

where σ is the random walk standard deviation at the time t

- Send in a pulse of mosquitoes
- Add global equations to calculate the outflow

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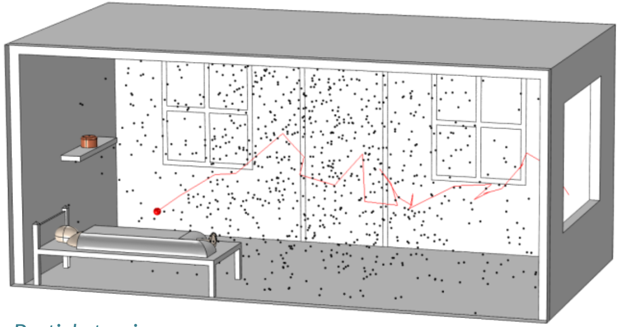
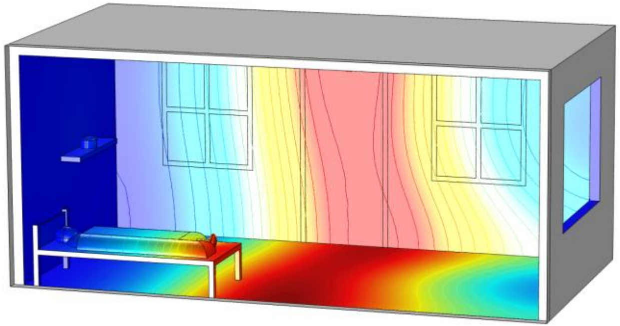
DEMO

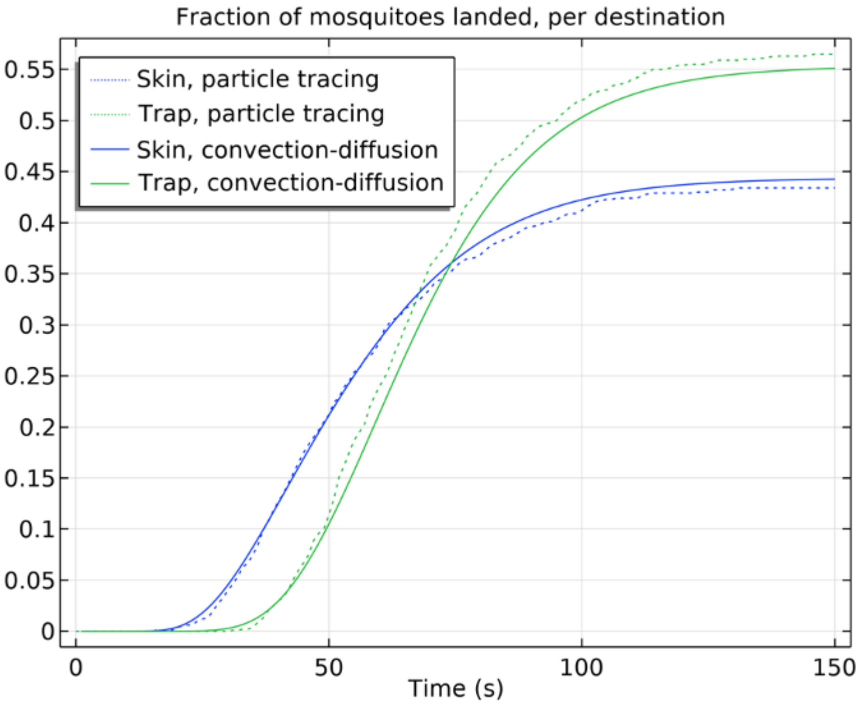
Mosquito Trap



Particle tracing.



Convection–diffusion.



Partial Differential Equation (PDE) Interfaces

Coefficient Form PDE

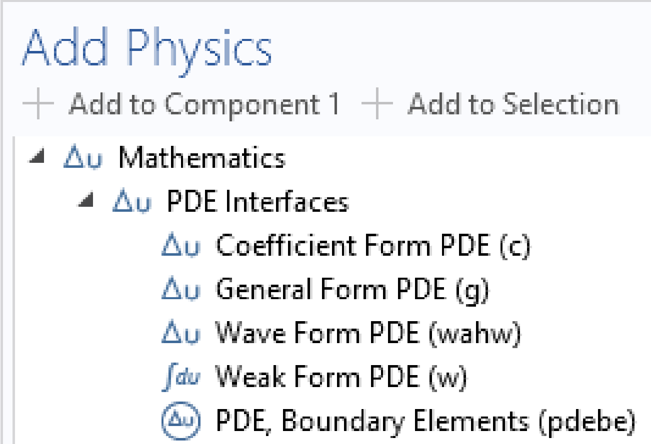
Comprehensive template for a PDE

$$e_a \frac{\partial^2 u}{\partial t^2} + d_a \frac{\partial u}{\partial t} + \nabla \cdot (-c\nabla u - \alpha u + \gamma) + \beta \cdot \nabla u + au = f$$

General Form PDE

Compact template, designed for conservation laws

$$e_a \frac{\partial^2 u}{\partial t^2} + d_a \frac{\partial u}{\partial t} + \nabla \cdot \Gamma = f$$



PDE Interfaces

- A physical equation can be mapped to a generic PDE by "coefficient matching"
- As an example, consider heat conduction:

$$\rho C_p \frac{\partial T}{\partial t} + \nabla \cdot (-k\nabla T) = Q$$

$$e_a \frac{\partial^2 u}{\partial t^2} + d_a \frac{\partial u}{\partial t} + \nabla \cdot (-c\nabla u - \alpha u + \gamma) + \beta \cdot \nabla u + au = f$$

$$\nabla = [\frac{\partial}{\partial x}, \frac{\partial}{\partial y}]$$

- This could be expressed using the *Coefficient Form PDE* by manually defining the coefficients

Further Resources to Get Started

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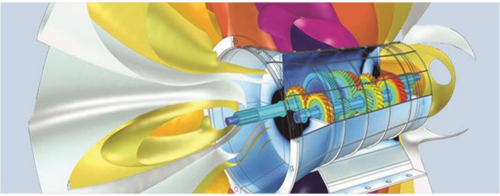
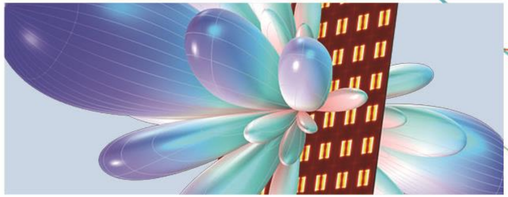
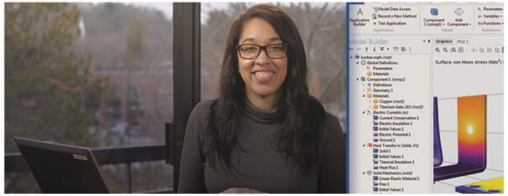


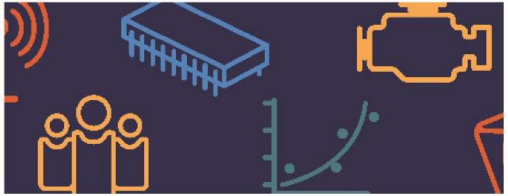
PRODUCT DOWNLOAD



MODELS & APPLICATIONS



LEARNING CENTER



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