OPTIMAL ELECTRICAL DESIGN OF SPHERICAL PHOTOVOLTAIC CELLS

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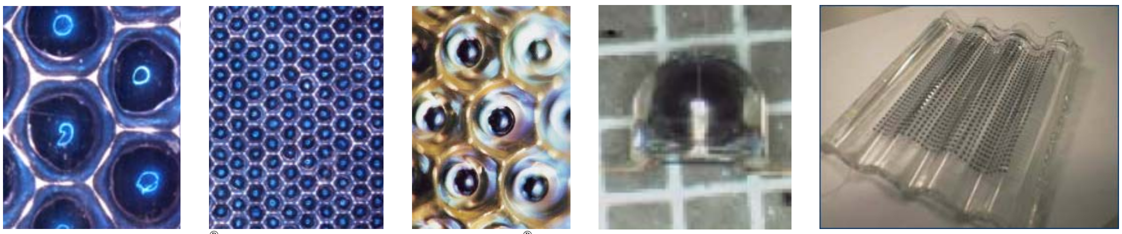
Abstract

In constructal theory, the optimal shape (geometry) and structure of natural and engineered systems is the outcome of their functionality and resources, and of the constraints to which they are subject. This paper reports the optimization results of the new generation of honeycomb spherical photovoltaic cells (SPVC) with respect to the series electrical resistance. It is assumed that the cells work under steady state conditions. The electrical potential (voltage) distribution is found by numerically integrating the mathematical model of the DC current distribution within the SPVC.

1. Introduction

Although the interest on Photovoltaic Cells (PVC) has increased in the recent years due to the energy crisis and the advance of the alternative energy sources, the recent shortages of high-grade silicon – used as raw material – may have a significant impact on the growth of the PV industry. Recently, novel spherical photovoltaic cell (SPVC) technologies were developed, for instance the Sphelar™ of Kyosemi (Fig. 1,*c*) that captures sunlight in all directions and increase its power generation capacity: it can minimize output fluctuations even under direct sunlight, and even when the angle of reflected incident light changes, as it can capture light as direct sunlight, as light diffused by clouds, and as light reflected from buildings [1]-[5].



a. Solar Power® array [3]. Honeycomb SPVCs. ***b.*** Fujipream® [4]. ***c.*** Spherical PVC. Wired SPVCs – Kyoto Semiconductor Corporation® [1]

Figure 1: Spherical solar cells.

Practically non-directive, SPVC are excellent in mechanical strength, and pose few restrictions on their mounting. Each sphere (bead) acts like an individual solar cell absorbing sunlight and converting it into electricity. The diameter of a SPVC should be small (0.2 – 2 mm; Fig. 1,*c*) in order to increase the proportion of the light reception surface area of the semiconductor crystal to its volume so as to raise the efficiency of the material [3]. The SPVC has a single spherical *p-n* junction (Fig. 2), and its maximum open voltage is the same as that of a larger flat junction type cell. The SPV cells have great potential because they are cheap, simple and fast processing, they can be assembled in flexible and lightweight modules and be integrated in new application possibilities.

The photoelectric current is driven, through the *n*-layer, to a high conductivity grid to the electrical terminals: this current path defines the *series resistance* of the SPVC, and it is responsible for the flattening of the current-voltage characteristic and a consequent loss of output power. It can be minimized by using good electrical contacts, surface layers of low resistance, and by optimizing the grid (collector) geometry.

The *honeycomb* technology (Fig. 1,*a*) comprises thousands of inexpensive tiny silicon spheres bonded between two thin layers of aluminum foil substrates, sealed on both sides by plastic. The aluminum layers give the material physical strength and act as electrical contacts: the front foil determines the spacing of the spheres and acts as the electrically negative contact to the outside of the spheres (n-type) and the back foil is the electrically positive contact to the core of the spheres (p-type).

The *wired* SPVC modules (Fig. 1,*b*) are produced in a variety of power needs ranging from an extremely small to a large power source – e.g., through connection of cells in series and parallel, with fine copper wire [1]. The mounting may be white resin reflection plate, with its surface covered with transparent resin. The electrode arrangement makes the cell non-directive, it can realize an even distribution of generated current, and facilitates a serial and parallel connection of the cell to the other.

The optimization reported here consists of minimizing the *sum* of the collector shadow and the series resistance (Joule) losses. Despite the many physical processes within the PVC [20] this optimization may be conducted separately [21]. Here, we are concerned with finding a collector grid design that leads to minimum series resistance, and our approach is based on the constructal theory [14]-[19], which addresses the following basic volume-to-point access problem [13], [16]: *given a finite size volume in which electrical current is generated at every point, which is connected by a small patch (terminal) located on its boundary, and a finite amount of high (electrical) conductivity material, determine the optimal distribution of high conductivity material through a given volume such that the highest voltage is minimized*.

Unlike the flat-surface PVC constructal optimization [13], the design goal here is to find the particular pattern of the PV spheres (beads) distribution on a high conductivity material foil (the cathode), or a wired network, that would connect the PV beads such that the series resistance of the cell is minimized. In what follows we present several types optimized geometries for honeycomb and wired SPVCs.

2. Mathematical Model

In what follows we assume that the SPVC operates under DC conditions and the electric field is derived from the electric potential, V. For the n-layer of the tiny SPVC (the emitter), either arrayed in the cathode honeycomb foil or wired through very thin conductor strips (the collector), the current flow is essentially 2D. The mathematical model is made of the following Laplace-Poisson equations

$$\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}+\frac{w'''}{\sigma_0}=0\text{, in the emitter} \tag{1a}$$

$$\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}=0\text{, in the collector} \tag{1b}$$

Here, w''' is the PV current source (assumed uniform), σ_p, σ_0 are the electrical conductivities of the collector and emitter, respectively. The current density and the voltage verify Ohm's law, $\mathbf{J}=-\sigma_{0,p}\nabla V$. The boundary is assumed electrically insulated (a Neumann homogeneous condition), except for the output port through which the current exits the cell (set at ground potential). Equations (1) are then non-dimensionalized by dividing the coordinates with the length scale (here, the radius of the SPVC, r_0), and the current source with its actual value, w'''. The voltage scale is then $V_0 = w''' r_0^2 / \sigma_0$. The electrical conductivities, σ_0 and σ_p, are scaled with that of the SPVC cathode, σ_0.

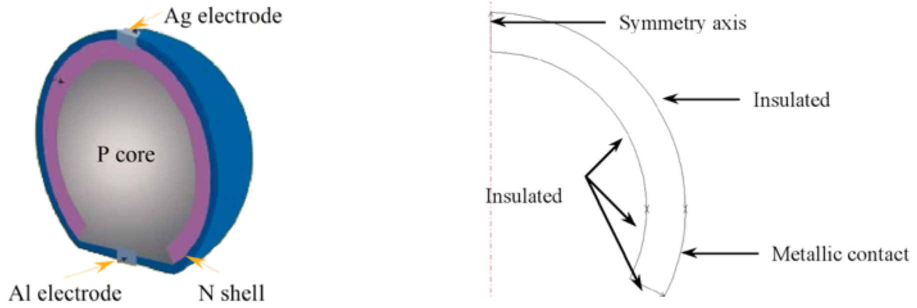
3. The honeycomb SPVC

The constructal problem posed by the honeycomb packaging differs from the fundamental problem [13] in the sense that the (current) sources are spread throughout a very good conducting material, which embeds the beads and cannot be distributed in a spanning tree structure. Further more, as the module edges act as paths of high conductive material they drain part of the current generated by the cells closer to the boundary. Another restriction posed by this design – if modeled at the spherical cells level – is the staggered arrangement.

Simplified 2D models for the spherical solar cell

First, a simplified 2D axial-symmetric model may be used to evaluate the current distribution through the n-layer (shell) of the SPV bead – Fig. 2,*b*. The following boundary conditions close the mathematical model made of the Laplace problem for the electrical potential:

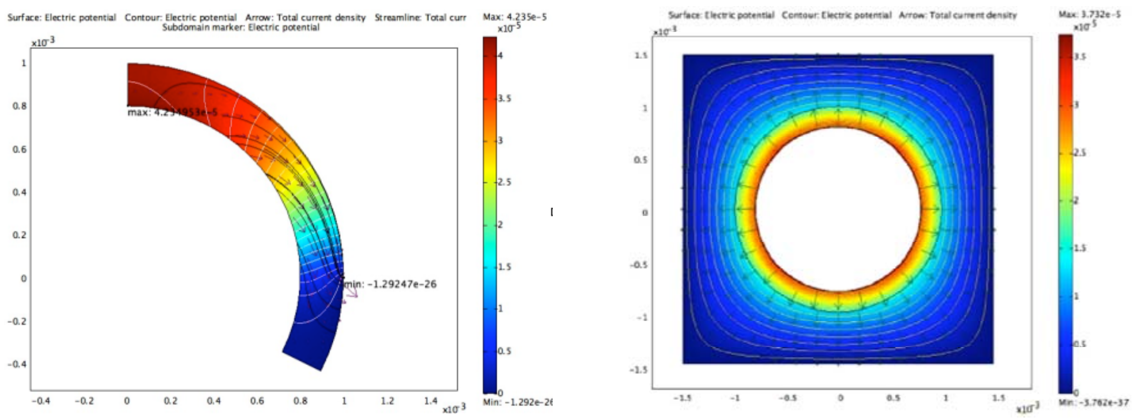
- For the inner surface of the shell (the actual *p-n* interface) a non-homogeneous Neumann condition is used to specify the photovoltaic current source.
- The normal component of the current density is zero at the outer surface of the bead.
- For the strip that represents the contact between the bead and the collector aluminum foil a Dirichlet condition is set, $V = 0$, because the excellent electrical conductivity of aluminum suggests an almost equipotential contact.



a. Cross sectional structure of a spherical SPVC [1]. ***b.*** A simplified 2D axial model.

Figure 2: The SPCV bead.

This model gives an estimate of the series resistance of the *n*-layer, witch is part of the global series resistance of the SPVC. Figure 3,*a* shows the electrical field (voltage surface color map and contour lines) and current flow (arrows and streamlines) within the shell obtained by COMSOL FEM analysis.



a. 2D Axial model – the electric field ***b.*** 2D Cartesian model – the electric field

Figure 3: The 2D equivalent models of a spheral solar cell – electric field spectrum.

The next step is to “flatten” the model, i.e. to recast it into a 2D Cartesian model that comprises also the collector (aluminum) “territory” of the bead: the *n* shell is projected to a circular crown that has the same series resistance as the actual spherical layer; the inner rim of the crown produces the same amount of current as the inner boundary of the spherical *n*-shell. The actual size of the aluminum patch that embeds the bead may be the object of an optimization problem. Figure 3,*b* depicts the voltage (surface color map and contour lines) and the current flow (arrows and streamlines) when the external boundary is set to ground. Of course, symmetry may be used to simplify the problem, but the numerical effort to solve this linear problem for the entire domain is not significant.

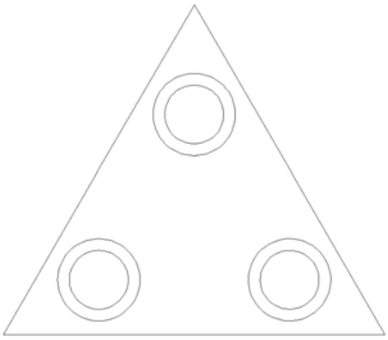
The 2D Cartesian model is further used to define the elemental cell of the constructal optimization sequence. The elemental cell may contain a number of SP beads, and it is the smallest entity, the construct or “brick” that is optimized for minimum series resistance: its shape and structure is essential to the shape and structure of higher order constructs in the constructal sequence.

The elemental cell

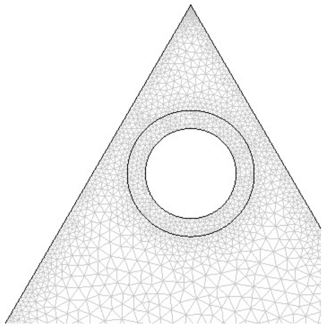
The first elemental cell design we propose (Fig. 4,*a*) is the simplest system that, by constructal growth, evolves into a staggered honeycomb SPVC module (Fig. 1,*a*). It is assumed that the photovoltaic current flows out the cell through the vertices, and that the edges are electrically insulated. The only degree of freedom allowed is the relative position of the beads along the principal axes of the triangular surface, between vertices and mass center.

The ratio between the peak voltage, wherever it occurs, and the total current produced by the SPV beads, defines the series resistance of the cell. Its inverse, the series conductance, is a quality factor (*QF*), a design quantity. As the series resistance is sensitive to the relative position of the beads, we carried out numerical experiments aimed at finding the layout that leads to its minimum.

Figure 4,*b* shows two meshes produced by the adaptive algorithm used to solve the conduction problem. The circular interior boundaries are current source, and the vertices are patched with tiny metallized electrodes: they are the current ports to the structure.



a. The elemental cell



b. The FEM mesh for elemental cell – detail.

Figure 4: The computational domain and the FEM mesh used in numerical simulation.

Figure 5 shows the electric field through the voltage (surface color map and contour lines) and the current density (arrows) in this optimization sequence.

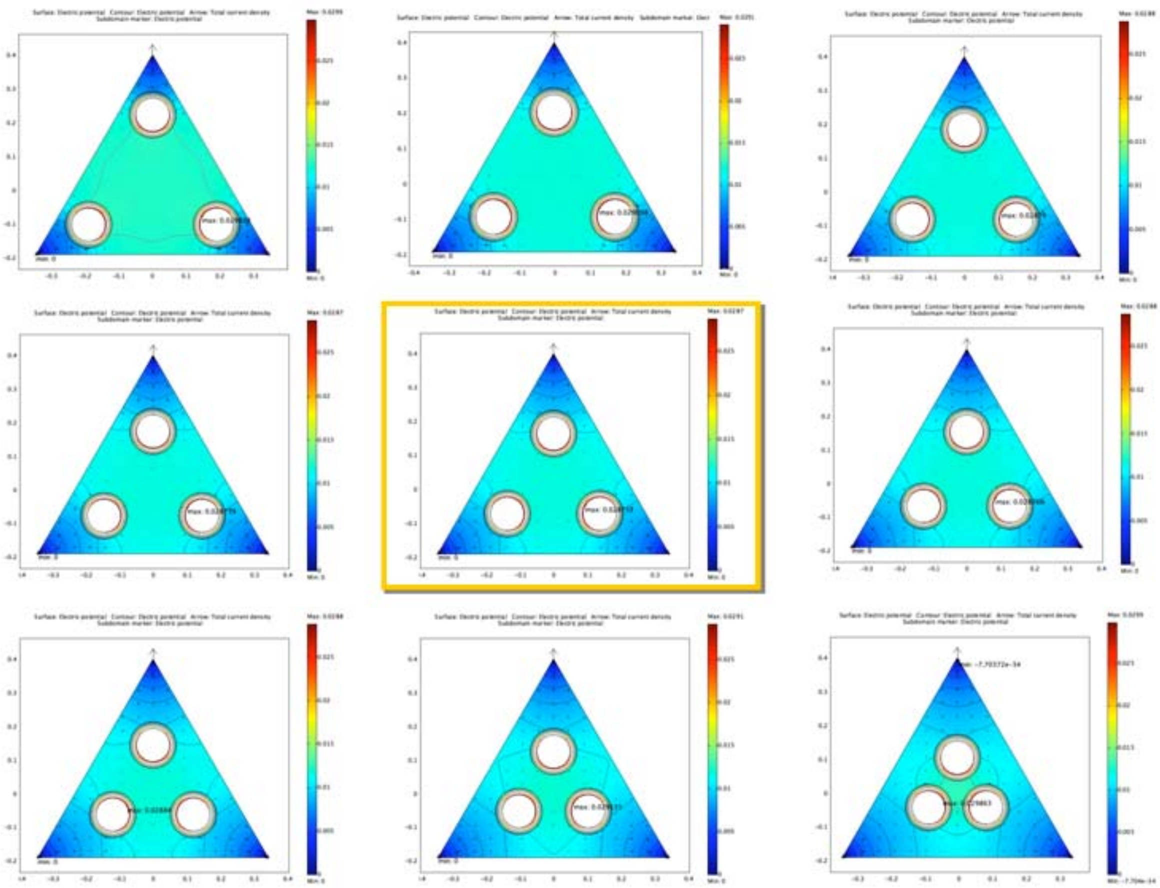


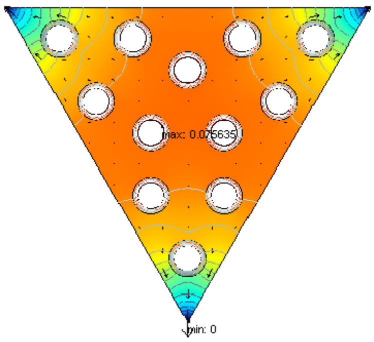
Figure 5: Constructal optimization of the elemental cell. The optimal cell is outlined.

By symmetry grounds, we conjecture that the optimal elemental cell layout has the same symmetry, and computed the quality factor for different position of the beads on the principal lines. The optimal design with highest *QF* (minimum, maximum voltage) is contoured.

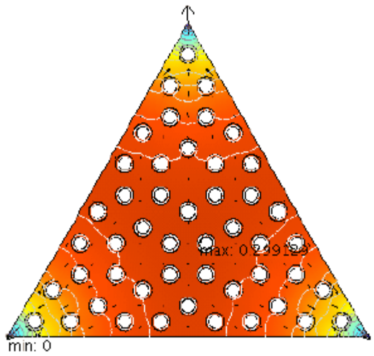
Higher order ensembles

Next, the optimized elemental cell is used to build higher order constructs, in a time arrow sequence: from lower to higher order structures. The first construct is obtained by mirroring the elemental cell with respect to its edges (Fig. 6,*a*).

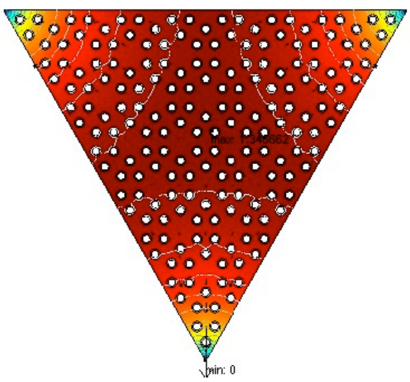
As this simple replication does not guarantee an optimum first order construct (minimum series resistance), numerical experiments – reported elsewhere [27] – were needed to validate the optimality of this design: we evaluated *QF* for different positions of the SPV beads along the principal lines of the first construct. There are countless layouts that might be considered, however we used the symmetry of this design to reduce the computational domain to 1/6 of its actual size, and the beads were displaced such as to preserve symmetry. The analysis confirmed the layout obtained by mirroring the elemental cell, and the reason is that the aluminum foil has a very good electrical conductivity as compared to the cells.



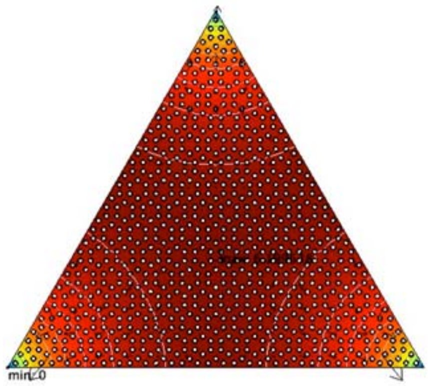
a. First order ensemble.



b. Second order ensemble.



c. Third order ensemble.



d. Fourth order ensemble.

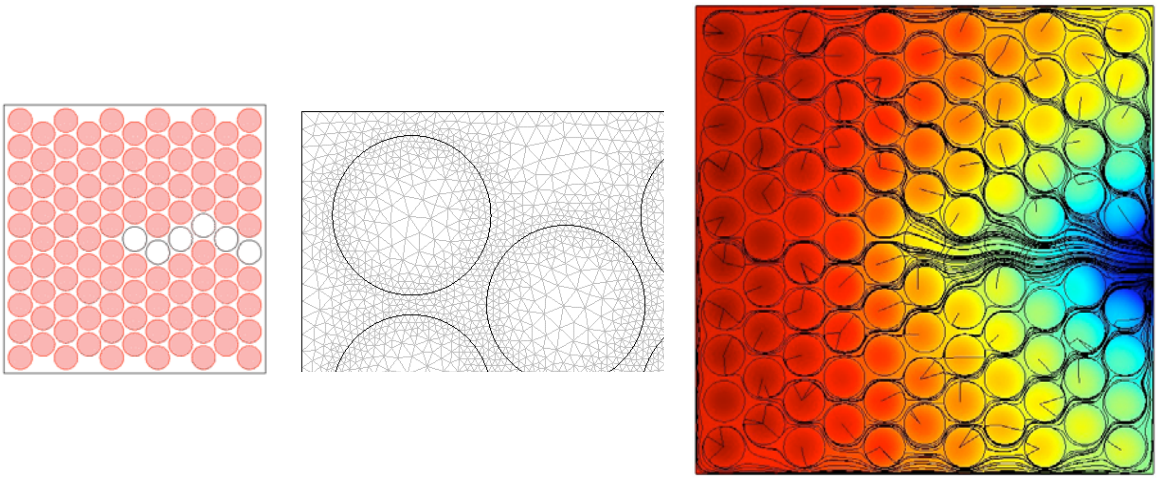
Figure 6: The first four higher order constructal ensembles – the electric field distribution

The mirroring technique may be pursued to generate ensembles of higher and higher order, propagating the triangular symmetry to the high orders constructs. Figure 6,*b*-*d* shows the second, third, and fourth order constructs and the electrical field calculated by numerical simulation and represented through the voltage (surface color map and contour lines) and the current density (arrows). Apparently, the inner regions are working at almost uniform voltage, and the vertices regions, used as electrical terminals, are areas of higher voltage gradient.

Remarkably, all constructs exhibit almost the same series resistance, which is a feature of constructal structures [16].

3. A quasi-constructal approach to the optimization of the honeycomb SPVC

In some cases when, by technological reasons (e.g., optimum spacing between spheres), it is not possible to depart from the regular honeycomb layout (Fig. 1,*a*), a quasi-constructal design may be utilized to improve the *QF*. For instance, a number of beads may be eliminated to produce supplementary high conductivity current paths. Figure 7 shows the elemental system where the dark colored disks (*a*) represent the SVPC, and the white background is the aluminum cathode. The port is seen at the boundary on the right. The white disks add to the white background to make the high conductivity path to the exit port. To pursue with a simpler, 2D analysis we assume that the SP beads are replaced by disks with uniform current generation. In this paper we briefly introduce the optimization results. A detailed study is found in [26].



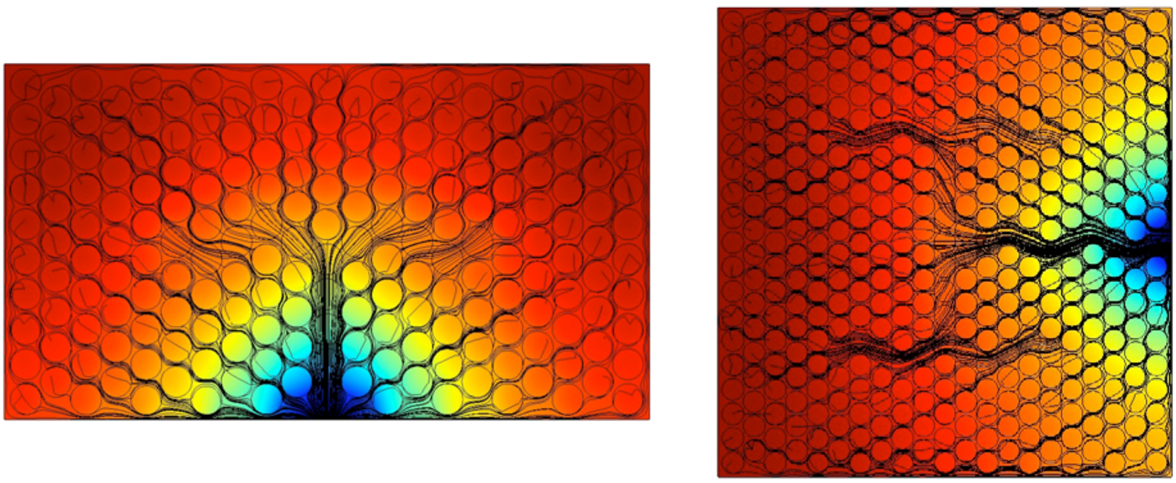
a. The elemental cell. ***b.*** The FEM mesh. ***c.*** The electrical field

Figure 7: The honeycomb elemental SPVC.

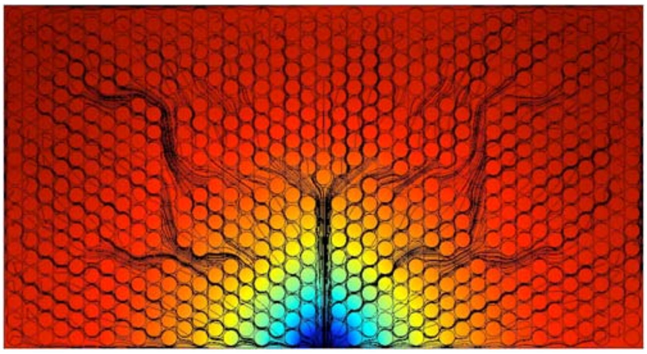
Figure 7,c presents the electric field by voltage surface map and current density streamlines. The hardest working point (highest voltage) is close to the good conducting path.

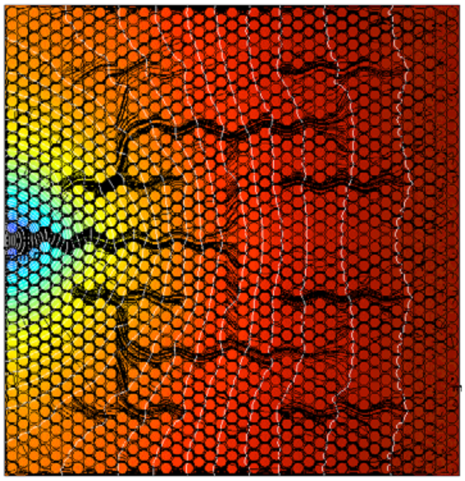
Next, the higher order ensembles are generated starting from the elemental cell. In order to pursue with this higher order structuring, each new ensemble results by combining two lower level optimized ensembles; in this process either one column or a row (depending on the order of the ensemble) is lost by partially overlapping the two constituent lower-level ensembles in order to preserve symmetry with an odd number of columns/rows. Also, the high conductivity path obtained by removing SPVCs preserves the same thickness in each ensemble. Depending on the ensemble order the path that connects the high conductivity tree to the port on the boundary may be either a straight or a saw-teeth-like strip (Fig. 8).

Figure 8 shows the voltage distribution on the first four higher order ensembles. As the order of the ensemble increases, the tree-like structure of the highly conductive material emerges: the tree is the flow architecture that provides the easiest (fastest, most direct) flow access between one point (source, or sink) and infinity of points (curve, area, or volume). Among other practical applications of tree-shaped flow architectures note the cooling of electronics [15], [23] and the flows through porous media [18], [19].



a. First order ensemble. ***b.*** Second order ensemble.





c. Third order ensemble. *d.* Fourth order ensemble

Figure 8: Higher order constructal ensembles – electrical field.

The series resistance for different conductivities aspect ratios, σ_p/σ_0, is reported in Fig. 9. The higher the conductivity ratio, the lower the voltage-drop on the module, hence the lower the losses by series resistance.

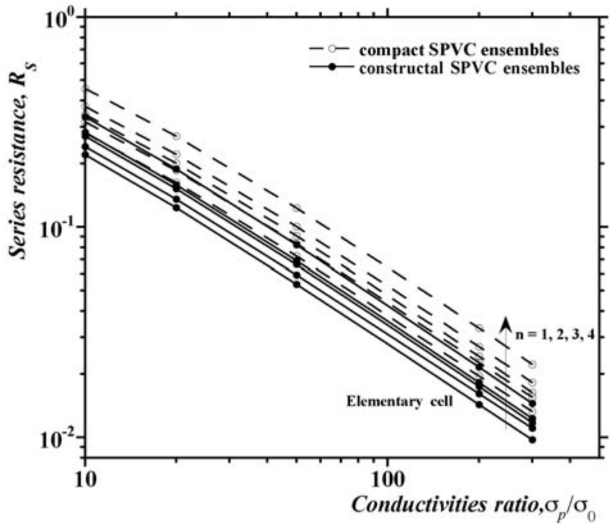


Figure 9: The series resistance (non-dimensional quantities).

Apparently, the optimized ensembles (continuous curves) have consistently lower series resistances than unstructured counterparts (dashed curves). An important feature exhibited by the optimized ensembles is that the (series) resistance does not vary with the ensemble order: this is an important feature of constructal structures that evidence their *scalability*. The departure recorded here is due to the quasi-constructal architecture that we adopted, imposed by the technological constrains that come with the honeycomb pattern and the spherical packaging.

Conclusions

The main conclusions drawn in this study are as follows:

- The constructal optimization in this paper aimed at minimizing the series resistance of the photovoltaic cell/module: the optimized assembly provides for the easiest access to the internal current. In the limit, the smallest acceptable elemental cell coincides with the smallest continuous system (cell) that is physically discernable.
- The optimization begins with the elementary cell and continues with higher order assembly in a "time-arrow" sequence (from small to large). This significantly differs from the fractal design, which proceeds from large to small – *ad infinitum* – through successive splitting.
- The constructal principle is deterministic and it is based on physical laws that describe the phenomena that occur in the systems under investigation.

- Although the optimizations presented here depart from the constructal sequence in several ways (e.g., the elemental cell is not actually minimized, the current path does not grow with the order of the ensemble for the honeycomb SPVC module), the main trends of the resulting ensembles are constructal: the analysis gains accuracy when the conductivities ratio $\sigma_p/\sigma_0 >> 1$; the ensembles present (almost) the same series resistance.
- Constructal minimization of R_S leads to designs that are not only optimal: they have also an attractive, *natural* appeal where the collector fingers are seen to evolve *naturally* into busbars. Depending on the particular shape and structure of the elemental system, the constructal design may match aesthetic criteria requested by architectural and design.

Acknowledgments

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